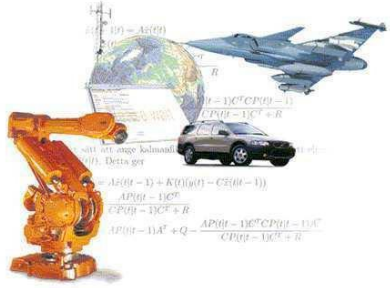


Identification of Industrial Robots

Robot Modeling and Control



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Types of Identification

3(29)

- Kinematic model
- Dynamic rigid body model
- Mechanical elasticities
- Nonlinearities in actuators and transmission
- Load dynamics
- Sensor and actuator disturbances (ripple)
- Thermal models
- Mechanical stress
- ...



Accurate robot models needed for:

- Control
- Mechanical design
- Performance simulation
- Diagnosis
- Supervision
- ...

System identification: Unknown model parameters are estimated from experimental data.



Outline

4(29)

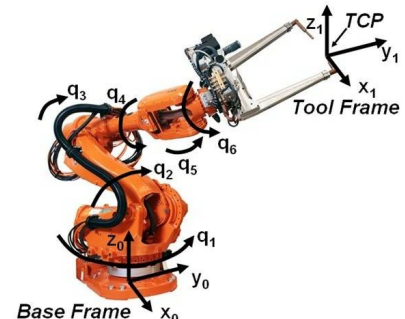
- Introduction
- Identification of Kinematic Parameters
- Identification of Rigid Body Dynamics
- Identification of Elasticities
- Conclusions



- Sciavicco, L. and Siciliano, B. *Modeling and Control of Robotic Manipulators*. Springer, 2000, pages 131–181.
- Wernholt, E. *Multivariable Frequency-Domain Identification of Industrial Robots*, PhD thesis, LiTH, 2007, pages 61–72.
<http://www.control.isy.liu.se/research/reports/Ph.D.Thesis/PhD1138.pdf>
- Olsson, R. *Identifiering av stelkroppsmodell för industrirobot*, Master's thesis, LiTH, 2005.
<http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-5535>
- Additional references can be found in the PhD thesis.



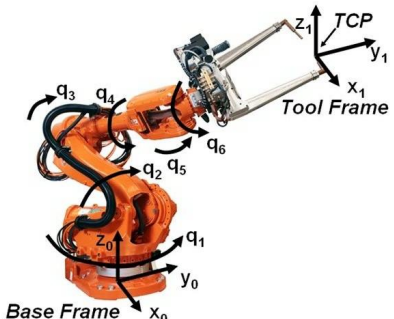
- Accuracy: Deviates a couple of millimeters with nominal kinematic model.



$$Z = \begin{bmatrix} \text{pos} \\ \text{ori} \end{bmatrix} = \Gamma_{kin}(q)$$



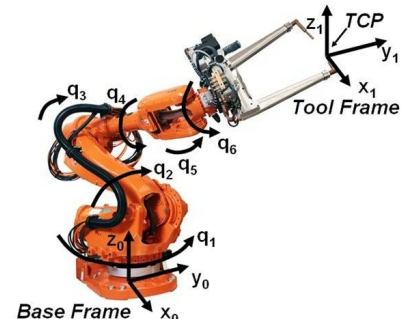
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- Repeatability: 0.1 mm for ABB IRB6400, that can handle 200 kg.



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- Accuracy: Deviates a couple of millimeters with nominal kinematic model.
- Repeatability: 0.1 mm for ABB IRB6400, that can handle 200 kg.
- Can obtain volumetric accuracy of 0.5 mm for a large robot by calibration.



$$Z = \begin{bmatrix} \text{pos} \\ \text{ori} \end{bmatrix} = \Gamma_{kin}(q)$$



- Taylor series expansion:

$$\Delta Z = Z - \Gamma_{kin}(q, \theta_{nom}) = \frac{\partial \Gamma_{kin}}{\partial \theta}(q, \theta_{nom}) \Delta \theta$$



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- Measure ΔZ in a number of locations, using e.g., a laser tracker, and solve for $\Delta \theta$ in least-squares sense.
- θ includes robot link parameters and parameters describing elastostatic effects (deflection due to gravity) and the mounting of the robot base and the end effector.



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Introduction

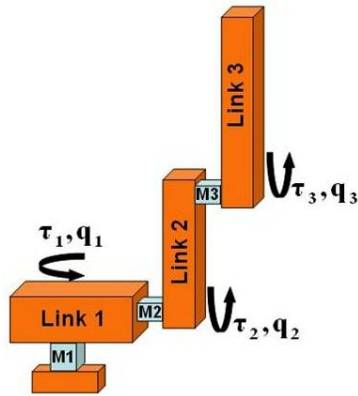
Identification of Kinematic Parameters

Identification of Rigid Body Dynamics

Identification of Elasticities

Conclusions





$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) + \tau_f(\dot{q}) = \tau$$



- The dynamics is *linear in the dynamic parameters*, often called the *standard inertial parameters*, θ_{rb} .
- Each link gives ten inertial parameters: body mass, mass location, and inertia matrix (six elements due to symmetry).



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- **Note:** This requires that the inertial parameters of each link are expressed w.r.t. a frame located in the joint, *not* the center of mass. Otherwise, not linear in mass location. (see Sciavicco and Siciliano, 2000)
- It is common to use a friction model

$$\tau_f(\dot{q}) = F_v \dot{q} + F_c \text{sign}(\dot{q})$$

which gives two additional parameters for each link, but still an expression that is linear in the parameters.



The robot dynamic model can then be rewritten as

$$H_{rb}(q, \dot{q}, \ddot{q})\theta_{rb} = \tau$$

or as the energy difference model

$$\Delta h_{rb}(q, \dot{q})\theta_{rb} = \Delta \mathcal{H}_{rb} = \mathcal{H}_{rb}(t_b) - \mathcal{H}_{rb}(t_a) = \int_{t_a}^{t_b} \tau^T \dot{q} dt$$

where $\theta_{rb} \in \mathbb{R}^{12n}$ is the parameter vector and n is the number of links. \mathcal{H}_{rb} is the total energy of the system.



Consider N samples of data,
 $Z^N = \{q(t_i), \dot{q}(t_i), \ddot{q}(t_i), \tau(t_i), i = 1, 2, \dots, N\}$:

$$\underbrace{\begin{bmatrix} H_{rb}(q(t_1), \dot{q}(t_1), \ddot{q}(t_1)) \\ H_{rb}(q(t_2), \dot{q}(t_2), \ddot{q}(t_2)) \\ \vdots \\ H_{rb}(q(t_N), \dot{q}(t_N), \ddot{q}(t_N)) \end{bmatrix}}_{\Phi} \theta_{rb} = \underbrace{\begin{bmatrix} \tau(t_1) \\ \tau(t_2) \\ \vdots \\ \tau(t_N) \end{bmatrix}}_Y$$



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- *Base parameters* θ_b – minimum number of parameters that characterize the dynamic model.
- Different approaches can be found in the literature. Most basic distinction is if the problem is solved using a numerical (SVD or QR factorization of Φ) or an analytical method.



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- *Base parameters* θ_b – minimum number of parameters that characterize the dynamic model.
- Different approaches can be found in the literature. Most basic distinction is if the problem is solved using a numerical (SVD or QR factorization of Φ) or an analytical method.
- $H_{rb}\theta_{rb} = \tau$ is replaced by

$$H_b(q, \dot{q}, \ddot{q})\theta_b = \tau$$



- The robot is moved along a trajectory and joint motion and torque are measured.



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- Finally the base parameters can be estimated, for example, by a *weighted least squares (WLS)* method

$$\hat{\theta}_b = \arg \min_{\theta_b} \frac{1}{2} (Y - \Phi_b \theta_b)^T W (Y - \Phi_b \theta_b) = (\Phi_b^T W \Phi_b)^{-1} \Phi_b^T W Y$$

- To obtain good results, experiment design is important, e.g., path as sum of sinusoids and maximize $\log \det(\Phi_b^T W \Phi_b)$. See, e.g., (Olsson, 2005).



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Introduction

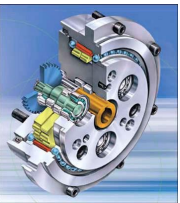
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Conclusions





- Transmission
- New structure (single bearings, etc.)
- Reduced weight ⇒ Increased elastic effects
- Mounting and equipment



Extended Flexible Joint Model

Modern robots require additional elastic effects to be modeled (due to weight reduction, single bearings, non-symmetric), e.g., by using the **Extended Flexible joint model**:

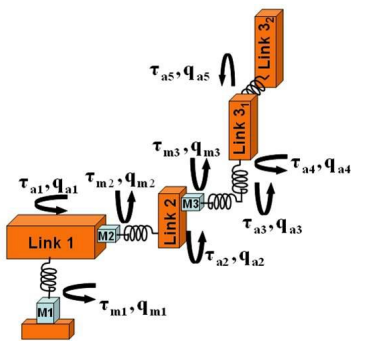
$$M_m \ddot{q}_m + \tau_{fm}(\dot{q}_m) + r_g \tau_g = \tau,$$

$$M_{ae}(q_a, q_e) \begin{bmatrix} \ddot{q}_a \\ \ddot{q}_e \end{bmatrix} + c_{ae}(q_a, q_e, \dot{q}_a, \dot{q}_e) + g_{ae}(q_a, q_e) = \begin{bmatrix} \tau_g \\ \tau_e \end{bmatrix}$$

$$k_g(r_g q_m - q_a) + d_g(r_g \dot{q}_m - \dot{q}_a) = \tau_g,$$

$$-k_e q_e - d_e \dot{q}_e = \tau_e,$$

(Moberg and Hanssen, 2007)

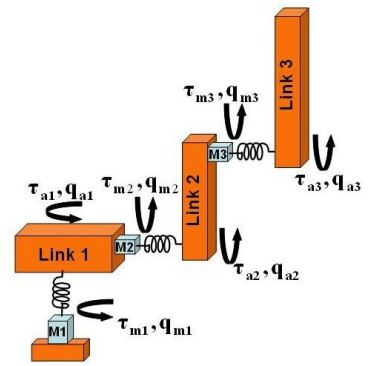


For many (traditional) robots, the joint flexibility is dominant, which motivates the **Flexible joint model**:

$$M_m \ddot{q}_m + \tau_{fm}(\dot{q}_m) + r_g \tau_g = \tau,$$

$$M_a(q_a) \ddot{q}_a + c_a(q_a, \dot{q}_a) + g_a(q_a) = \tau_g,$$

$$k_g(r_g q_m - q_a) + d_g(r_g \dot{q}_m - \dot{q}_a) = \tau_g,$$



Rigid vs. Elastic Body Dynamics Id.

- Identification of mechanical elasticities is more complex than identification of rigid body dynamics.

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- (This could partly be solved by adding expensive sensors.)



Identification of Mechanical Elasticities

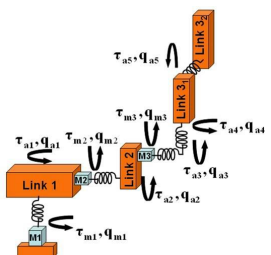
Given: Kinematic and rigid body models from a priori knowledge or previous identification experiments.

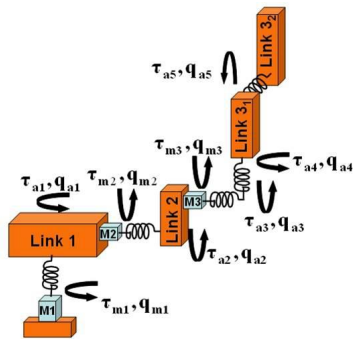


Identification of Mechanical Elasticities

Given: Kinematic and rigid body models from a priori knowledge or previous identification experiments.

Aim: Identify springs and dampers





- Accurate models are needed
- Multivariable
- Nonlinear
- Unstable
- Highly resonant
- Data collected in closed loop
- Normally only motor angle measurements available
- Difficult disturbance properties

$$\dot{x}(t) = f(x(t), u(t), \theta),$$

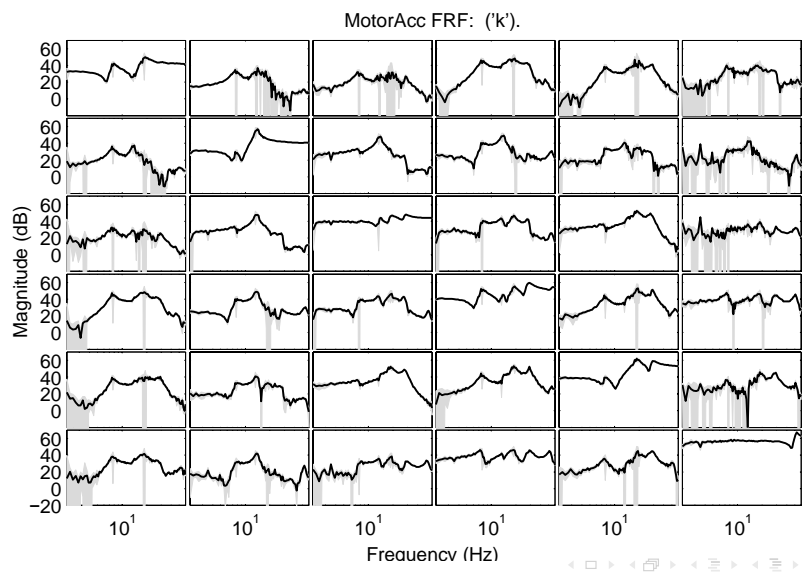
$$y(t) = h(x(t), u(t), \theta),$$



1. Estimate **nonparametric FRFs** in Q operating points

$$\hat{G}^{(i)}(\omega_k), \quad i = 1, \dots, Q$$

► Pros & Cons



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2. Linearize the nonlinear parametric robot model in each position, resulting in Q **parametric FRFs** ► Details

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$$G^{(i)}(\omega_k, \theta), \quad i = 1, \dots, Q$$

3. Obtain $\hat{\theta}$ by minimizing the discrepancy between the parametric and nonparametric FRFs.

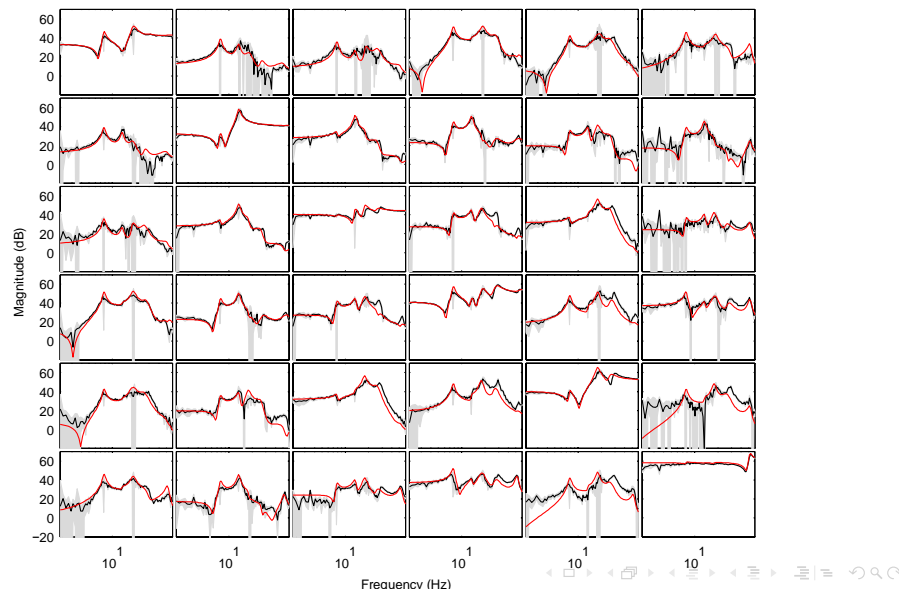
▶ Pros & Cons

$$\hat{\theta} = \arg \min_{\theta} \sum_{i=1}^Q \sum_{k=1}^{N_f} [\mathcal{E}^{(i)}(k, \theta)]^H [\Lambda^{(i)}(k)]^{-1} \mathcal{E}^{(i)}(k, \theta)$$

with

$$\mathcal{E}^{(i)}(k, \theta) = \log \text{vec}(\widehat{G}^{(i)}(\omega_k)) - \log \text{vec}(G^{(i)}(\omega_k, \theta))$$

and $\Lambda^{(i)}(k)$ a Hermitian weighting matrix.



- Identification of industrial robots varies depending on the model (kinematic, rigid body dynamics, elasticities, ...)
- Rigid body dynamics can be identified using linear regression techniques.
- Identification of elastic effects is quite challenging due to: multivariable, nonlinear, unstable, resonant, high accuracy required,

Frequency-domain

- + Data compression
- + Unstable systems ok
- + Easy to validate that resonances are captured
- + Frequency-domain requirements easily handled
- Biased estimates due to nonlinearities and closed loop
- Linear approximations might be inaccurate, e.g., for Coulomb friction.

Time-domain

- + No approximations
- Large data sets
- Numerical problems
- Stable predictor (nonlinear observer)

[Return](#)

For each operating point i and θ value do:

1. Calculate the stationary point $(x_0^{(i)}(\theta), u_0^{(i)}(\theta))$.
2. Linearize the nonlinear system.
3. Obtain the continuous-time transfer function as

$$G_c^{(i)}(s, \theta) = C^{(i)}(\theta)(sI - A^{(i)}(\theta))^{-1}B^{(i)}(\theta) + D^{(i)}(\theta).$$

4. Convert the continuous-time FRF $G_c^{(i)}(j\omega, \theta)$ to discrete time (assuming zero-order hold):

$$G_{T_s}^{(i)}(e^{j\omega T_s}, \theta) \approx \frac{1 - e^{-j\omega T_s}}{j\omega T_s} G_c^{(i)}(j\omega, \theta).$$

$(G^{(i)}(\omega, \theta))$ is a short notation for $G_{T_s}^{(i)}(e^{j\omega T_s}, \theta)$

[Id. Proc.](#)