Combining kernel-based methods and the EM method for structured system identification

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Outline

- Kernel-based linear system identification
- Handling input uncertainties
- Handling outliers

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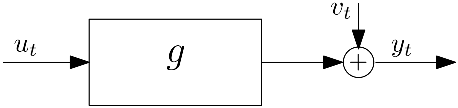
A standard system identification problem



Model in time domain: $y_t = (g * u)_t + v_t$

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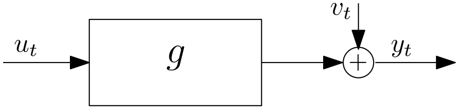

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Assumptions

- g BIBO stable, unknown order
- v_t white Gaussian (unknown variance σ^2)

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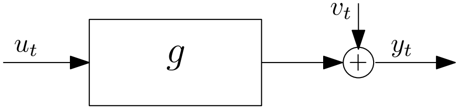

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Linear regression problem: $y = Ug + v$

A Bayesian point of view

g **is a Gaussian vector:** $g \sim \mathcal{N}(0, \lambda K_\beta)$

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Stable spline kernel

[Pillonetto & De Nicolao,'10],[Chen, Ohlsson, Ljung,'12]

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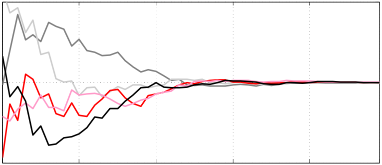

BIBO stable realizations

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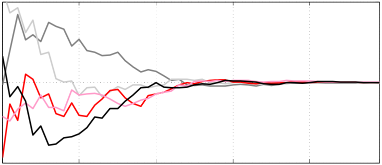

BIBO stable realizations

Two hyperparameters to be chosen

1. λ: amplitude of the impulse response (> 0)
2. β: related to the decay rate of g

Estimation of the impulse response

Exploiting joint Gaussianity between g and y...

$$p(g|y) = \mathcal{N}(Cy,\, P)$$

$$P = \left(\frac{U^T U}{\sigma^2} + (\lambda K_\beta)^{-1} \right)^{-1} \qquad C = P \frac{U^T}{\sigma^2}$$

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MMSE estimator of g:

$$\hat{g} = \mathbb{E}[g|y] = C(\theta) y$$

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... but it depends on the parameter vector $\theta = [\lambda,\ \beta,\ \sigma^2]$

Parameter selection

Empirical Bayes approach *(Pillonetto, Chiuso, De Nicolao, '11)*

- Marginal distribution of y:

$$p(y) = \int p(y, g) dg$$

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Can we come up with other optimization strategies?

Optimizing via EM

The EM method

1. Define "missing data": our unknown system g
2. Introduce the complete likelihood: $L(y, g; \theta) := \log p(y, g; \theta)$

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Convergence properties

[Dempster et al., 1977]: $\hat{\theta}^{(k)} \longrightarrow$ local solution of $\arg\max \log p(y;\theta)$

EM in our problem

At each iteration, compute

$$\hat{g}^{(k)} = C(\theta^{(k)})y \qquad P^{(k)} = \mathrm{Cov}\left[g^{(k)}\right]$$

Recall: $\theta = [\lambda,\, \beta,\, \sigma^2]$

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- $\hat{\lambda}^{(k+1)} = \frac{1}{n}\mathrm{tr}\left[K_{\hat{\beta}^{(k)}}^{-1}(P^{(k)} + \hat{g}^{(k)}\hat{g}^{(k)T})\right]$ $\hookleftarrow$ closed-form

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- $\hat{\sigma}^{2,(k+1)} = \frac{1}{N}\left(\|y - U\hat{g}^{(k)}\|^2 + \mathrm{tr}\left\{UP^{(k)}U^T\right\}\right)$ ↩ closed-form

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Simple updates... but is it really worth it?

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Input uncertainties

Assumption

$$u = \begin{bmatrix} u_1 & \dots & u_N \end{bmatrix}^T \quad \longrightarrow u = Hx \quad , \quad x \in \mathbb{R}^p$$

H known matrix, x unknown vector $\hookleftarrow$ needs to be estimated

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How to estimate x?

Include x in the ML optimization: $\theta := [x^T \ \lambda \ \beta \ \sigma^2]$

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EM estimation

Given the k-th iteration parameter guess $\hat{\theta}^{(k)}$, compute

- $\hat{x}^{(k+1)} = (A^{(k)})^{-1} b^{(k)}$ ← $A^{(k)}$ and $b^{(k)}$ easily computed
- $\hat{\lambda}^{(k+1)}$, $\hat{\beta}^{(k+1)}$, $\hat{\sigma}^{2,(k+1)}$ as before ← use $\hat{u}^{(k)} = H\hat{x}^{(k)}$

Input uncertainties (2)

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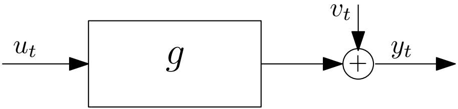
Examples

- Semi-blind system identification
- Hammerstein system identification
- Estimation of initial conditions

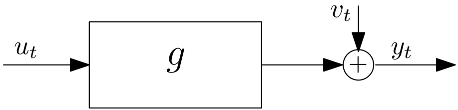
Semi-blind system identification



- Same setup as before...
- but no measurements of u_t

G. Bottegal, R.S. Risuleo, H. Hjalmarsson.
Blind system identification using kernel-based methods. *IFAC SysId 2015*

Semi-blind system identification



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Assumption

u belongs to a p dimensional subspace (semi-blind)

$\hookrightarrow$ We know H such that $u = Hx$

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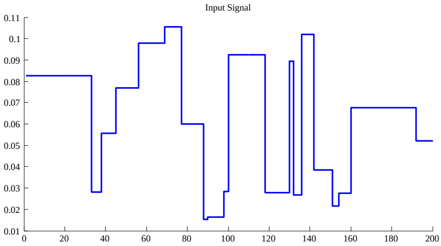
Input models

Example: u piecewise constant with known switching instants



$$H = \begin{bmatrix} \mathbf{1}_{T_1} & & & \\ & \mathbf{1}_{T_2 - T_1} & & \\ & & \ddots & \\ & & & \mathbf{1}_{T_p - T_{p-1}} \end{bmatrix}$$

$x =$ input levels

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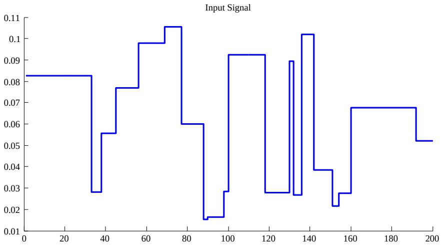

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$x =$ input levels

Applications:

- Occupancy estimation from CO_2 measurements

  (A. Ebadat et al. *Blind identification strategies for room occupancy estimation.* ECC '15)

- Non-intrusive load monitoring

Identification procedure

Identifiability

Any pair $\left(g,\, \frac{1}{\alpha}u\right)$ explains the data equally well!

$\hookrightarrow$ Assume $\|g\|_2 = 1$ and $g_1 > 0$

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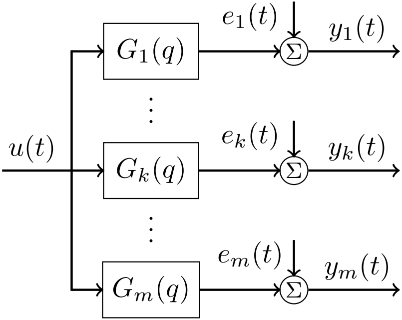
Identification

- Fix $\lambda = 1$ and repeat EM steps until convergence:
  1. update $\hat{x}$
  2. update $\hat{\sigma}^2$
  3. update $\hat{\beta}$
- Compute $\hat{u} = H\hat{x}$
- Compute $\hat{g} = C(\hat{\theta})y$ and normalize

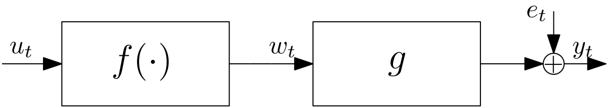
Full blind system identification



SIMO system

- *Enough* subsystems?
  $\hookrightarrow$ Full blind system identification ($H = I$)
- Noise spatial correlation can be included

Hammerstein system identification



$$w_t = f(u_t)$$
$$y_t = \sum_{k=1}^{\infty} g_k w_{t-k} + e_t$$

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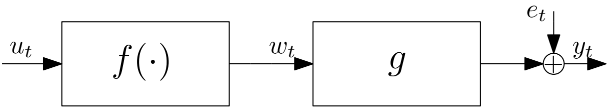
Hammerstein system identification



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Modeling approach

- p basis functions for f: $f(u_t) = \sum_{i=1}^{p} x_i h_i(u_t)$ $\hookleftarrow$ only h_i's known
- kernel-based model of g, with $\|g\|_2 = 1, g_1 > 0$ $\hookleftarrow$ for identifiability

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R.S. Risuleo, G. Bottegal, H. Hjalmarsson.

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Identification with EM

How to cast Hammerstein system identification into our framework?

$$w = \begin{bmatrix} \sum x_i h_i(u_1) \\ \vdots \\ \sum x_i h_i(u_N) \end{bmatrix} = Hx$$

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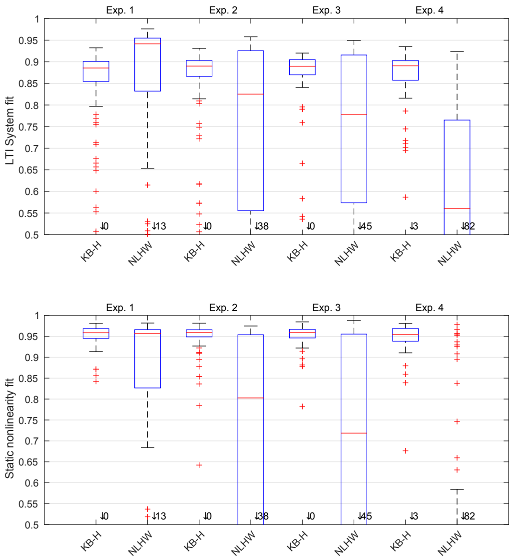
- Identification: same algorithm as semi-blind system identification
- Extension: nonparametric models of f (see next poster session)

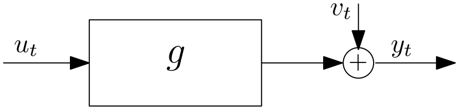
Simulations



Features

- $N = 500$
- $n = 100$
- SNR $= 10$
- LTI orders $= [4,8,10,20]$
- 7th order polynomial

Estimating the initial conditions



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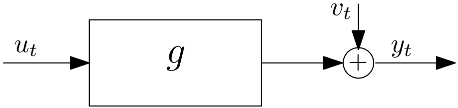
IEEE CDC 2015

Estimating the initial conditions



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Recall: $y = Ug + v$

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Initial conditions

The matrix U depends on unavailable input samples!

$$U := \begin{bmatrix} u_0 & u_{-1} & u_{-2} & & u_{-n+1} \\ u_1 & u_0 & u_{-1} & & u_{-n+2} \\ u_2 & u_1 & u_0 & \cdots & u_{-n+3} \\ \vdots & \vdots & \vdots & & \vdots \\ u_{N-1} & u_{N-2} & u_{N-3} & & u_{N-n} \end{bmatrix}$$

Solutions?

How do we cope with initial conditions?

Initial conditions

The matrix U depends on unavailable input samples!

$$U := \begin{bmatrix} & & & & \\ & & & & \\ u_{n-1} & u_{n-2} & u_{n-3} & \cdots & u_0 \\ \vdots & \vdots & \vdots & & \vdots \\ u_{N-1} & u_{N-2} & u_{N-3} & \cdots & u_{N-n} \end{bmatrix}$$

Solutions?

Truncation $\hookleftarrow$ throw away n measurements!

Initial conditions

The matrix U depends on unavailable input samples!

$$U := \begin{bmatrix} u_0 & 0 & 0 & & 0 \\ u_1 & u_0 & 0 & & 0 \\ u_2 & u_1 & u_0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ u_{N-1} & u_{N-2} & u_{N-3} & & u_{N-n} \end{bmatrix}$$

Solutions?

Set to 0 $\hookleftarrow$ may be very inaccurate...

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Solutions?

Estimate them from data!

EM estimation

C.I. estimation + system identification

- Just set $x = [u_{-1} \; \ldots \; u_{-n+1}]$ $\qquad (H = I)$
- Apply EM-based identification procedure!

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N	150	250	400
Truncation	42.01	59.40	62.96
Zeros	51.69	61.15	63.18
EM-based	55.69	62.77	64.45
Oracle	57.31	63.78	64.95

Features

$n = 100$, $SNR = 20$, $u =$ ARMA process, system order $= 40$

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Outliers in system identification

Why should we contrast outliers?

G. Bottegal, A.Y Aravkin, H. Hjalmarsson, G. Pillonetto.
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Automatica (provisionally accepted)

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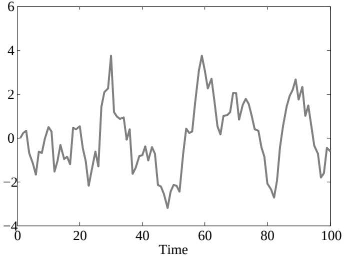
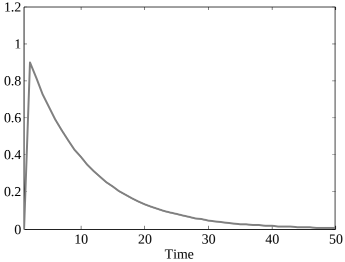


True impulse response



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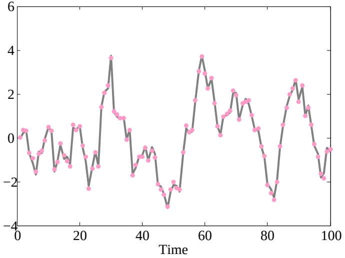
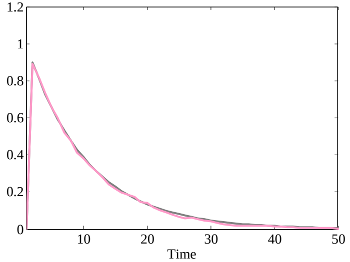


Estimated impulse response



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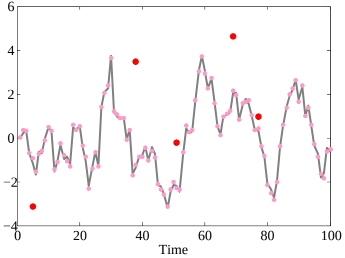
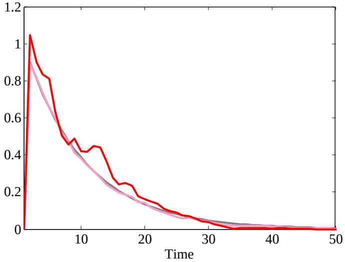
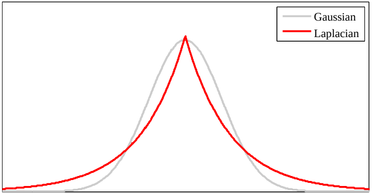


Estimated impulse response



G. Bottegal, A.Y Aravkin, H. Hjalmarsson, G. Pillonetto.
Robust EM kernel-based methods for linear system identification.
Automatica (provisionally accepted)

How to model outliers?



Long tailed random variables

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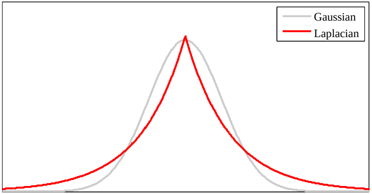

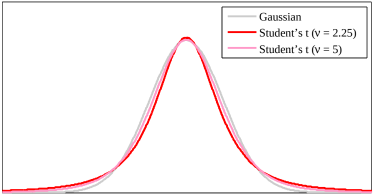

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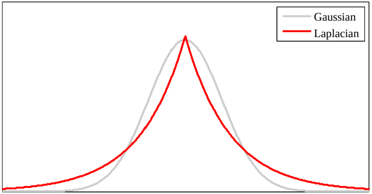

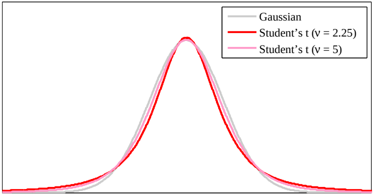

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Long tails $\longrightarrow$ Outliers are expected in this model

Handling non-Gaussianity

v_t non-Gaussian in our model $\longrightarrow$ No closed-form of ML and $\hat{g}$

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Need to estimate the new parameters τ_t

MAP estimate of parameters

- $\theta := [\lambda \ \beta \ \tau_1 \ \ldots \ \tau_N]$
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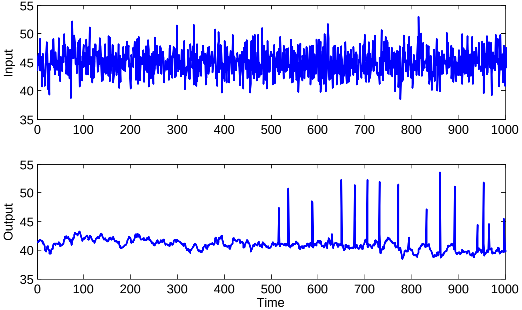
EM solution of MAP problem

- Update $\hat{\lambda}^{(k+1)}$ and $\hat{\beta}^{(k+1)}$ as usual
- Update rule for each τ_t is
  1. Laplacian case: $\hat{\tau}_t^{(k+1)} = \frac{\sigma^2}{4}\left(\sqrt{1+\frac{8\hat{\alpha}_t^{(k)}}{\sigma^2}}-1\right)$
  2. Student's t case: $\hat{\tau}_t^{(k+1)} = \frac{\hat{\alpha}_t^{(k)}+(\nu-2)\sigma^2}{\nu+3}$ ($\hat{\alpha}_t^{(k)}$ *known*)

A real experiment



Features

- Input: Voltage to pump
- Output: Water tank level
- Outliers due to pressure perturbation
- Identification using second part of data

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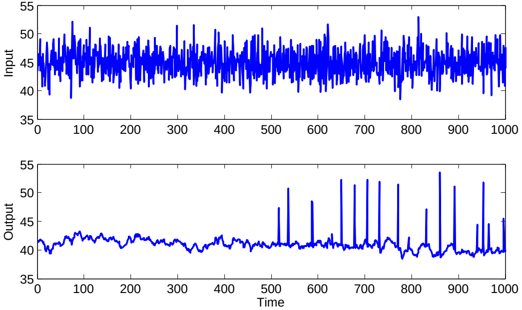

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Method	Fit % on first part of data
Student	67.40
Laplace	51.81
Gaussian	41.49
Gauss. (estimated using test set)	70.06

Conclusions

Starting point: kernel-based linear system identification
$\hookrightarrow$ requires tuning of some (hyper-)parameters

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Our contribution

- We can select parameters via ML+EM
- Nice iterative method in case of input uncertainties:
  - Blind system identification
  - Hammerstein systems
  - Estimation of initial conditions
- Nice iterative method for outlier robust system identification

Lots of questions!

Open questions

- Combine input uncertainties with robust technique?
- Compare with other iterative gradient-based methods?
- Other challenging problems (Wiener, networks, EIV,...)?
- Convergence rate?
- ...

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A special thank goes to...

- Riccardo S. Risuleo
- Håkan Hjalmarsson
- Gianluigi Pillonetto

Combining kernel-based methods and the EM method for structured system identification

Giulio Bottegal

KTH - Royal Institute of Technology (Sweden)

September 21, 2015

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