Homework 2

Networked Control and Multi-Agent Systems

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1.

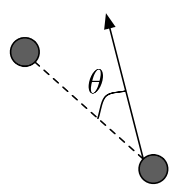
Consider a directed cycle graph of planar agents, where agent 1 can see agent 2, agent 2 can see agent 3, and so forth. Instead of the agents "aiming" at each other, let them have a certain degree of offset in their aim, i.e.,

$$\dot{x}_i = R(\theta)(x_{i+1} - x_i),\ i = 1, \ldots, N-1, \quad \dot{x}_N = R(\theta)(x_1 - x_N),$$

where $R(\theta)$ is the rotation matrix

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix},$$

as shown below.



a

Show that if the offset angle θ is given by

$$\theta = \frac{\pi}{N}$$

and the agents are initially placed evenly spaced on a circle, then they execute a circular motion (so-called cyclic pursuit).

b

What do you think would happen if $\theta > \pi/N$? What if $\theta < \pi/N$?

2.

Given an undirected graph $G = (V, E)$. An edge tension function is in general given by

$$\mathcal{E}(x) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\mathcal{E}_{ij}(x_i, x_j),$$

with $\mathcal{E}_{ij}(x_i, x_j) \neq 0$ only when $(i, j) \in E$.

The control law one would obtain from this is

$$\dot{x} = -\frac{\partial \mathcal{E}^T}{\partial x} \;\Rightarrow\; \dot{x}_i = -\sum_{j \in N_i} \frac{\partial \mathcal{E}_{ij}^T}{\partial x_i}.$$

a

Let (where we assume that $(i,j) \in E$ – otherwise $\mathcal{E}_{ij} = 0$)

$$\mathcal{E}_{ij} = (\|x_i - x_j\| - d_{ij})^2,$$

where d_{ij} is the desired distance between agents i and j.

What is $\dot{x}_i$? What will the system do under this choice of control law?

b

Same question as in 3a but with

$$\mathcal{E}_{ij} = \|x_i - x_j\|^2 - d_{ij}^2.$$

3.

One way of achieving translationally-invariant formations is to let the desired position for agent i be y_i, and to run the control protocol

$$\dot{x}_i = -\sum_{j \in N_i} ((x_i - x_j) - (y_i - y_j)).$$

Now, consider two connected agents on the line. Assume that there is some confusion about where the target positions really are. In particular, let agent 1 run the above protocol with $y_1 = -1$ and $y_2 = 1$. At the same time, agent 2 runs the protocol with $y_1 = 0$ and $y_2 = -3$.

What happens to $x_1(t), x_2(t)$, and $x_1(t) - x_2(t)$ as $t \to \infty$?

4.

In a leader-follower network, we typically let the followers be attracted to the leaders. But, if they are repelled by the leaders instead, we would get

$$\dot{x}_i = \sum_{j \in N_i} s_j (x_j - x_i),$$

where $s_j = 1$ is agent j is a follower and $s_j = -1$ if j is a leader.

For the graph K_3 with two followers and one repelling leader, is it possible for the leader to move in such a way that it prevents the two followers from meeting?

5.

We have seen examples of proximity graphs, i.e. graphs whose edges are geometrically defined. For example, a Δ-disk graph is a proximity graph $V \times E$ such that $(v_i, v_j) \in E \;\Leftrightarrow \|x_i - x_j\| \leq \Delta$, where $x_i \in \mathbb{R}^d$, $i = 1, \ldots, N$ is the state of robot i. In this question, we will be exploring another type of proximity graph, namely the *wedge graph*.

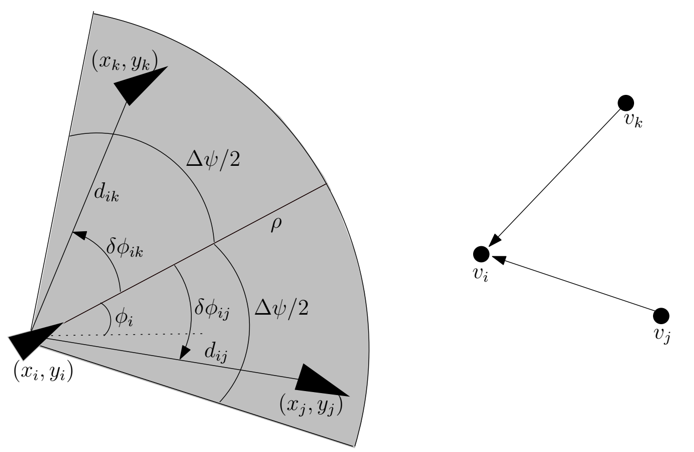
Assume that instead of single integrator dynamics, the agents' dynamics are defined as unicycle robots, i.e.

$$\dot{x}_i = v_i \cos \phi_i$$
$$\dot{y}_i = v_i \sin \phi_i$$
$$\dot{\phi}_i = \omega_i.$$

Here (x_i, y_i) is the position of robot i, while ϕ_i is its orientation. Moreover, v_i and ω_i are the translational and rotational velocities, which are the controlled inputs.

Now, assume that such a robot is equipped with a rigidly mounted camera, facing in the forward direction. This gives rise to a directed *wedge-graph*, as seen in the figure below. For such a setup, if robot j is visible from robot i, the avialable information is $d_{ij} = \|(x_i, y_i)^T - (x_j, y_j)^T\|$ (distance between agents) and $\delta\phi_{ij}$ (relative inter-agent angle) as per the figure below. (In fact, use the notation given in the figure.)

Explain how you whould try and solve the rendezvous problem for such a system. (Note: I don't need proofs, but I do need a discussion about the choices that you make.)



6.

Given a scale-invariant triangular formation

$$\dot{x}_i = -\sum_{j=1}^{3}(\|x_i - x_j\| - \alpha_i K)(x_i - x_j) + v, \ \ i = 1, 2, 3,$$

where K is the nominal, desired inter-agent distance, v is the general direction in which the formation is moving, and α_i is the scale parameter applied by agent i.

Now consider the situation below in which the three agents are to squeeze through a narrow opening (the gray areas correspond to obstacles). Discuss how you would go about selecting appropriate α_i's in a decentralized manner; both when communications are possible and when they are not. (You can always assume that you can measure the relative displacements.)