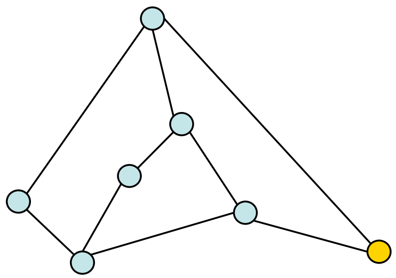
SESSION 3
CONTROL OF ROBOT TEAMS

Leader (Anchor) Nodes

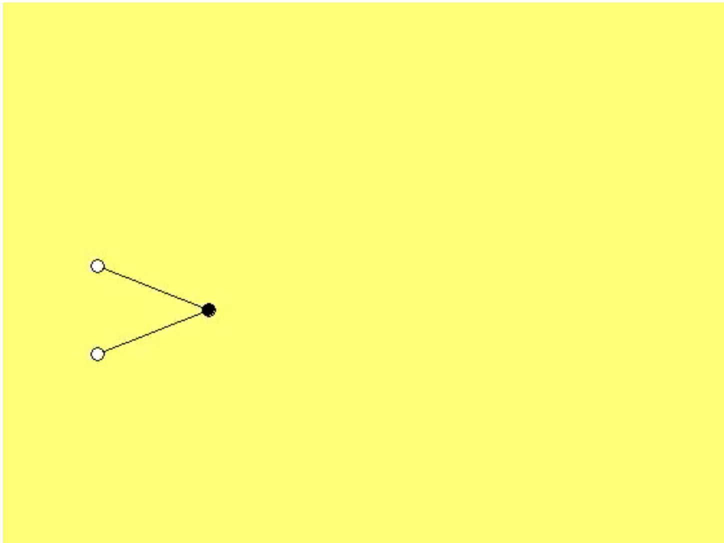
- **Key idea**: Let some subset of the agents act as control inputs and let the rest run some cohesion ensuring control protocol



A Mood-Picture

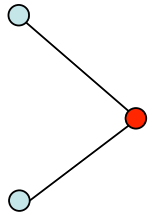
Graph-Based Controllability?

- We would like to be able to determine controllability properties of these systems directly from the graph topology



- For this we need to tap into the world of algebraic graph-theory.
- But first, some illustrative examples

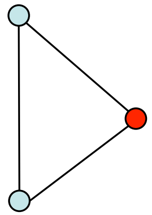
Some Examples



Not controllable!

Why?

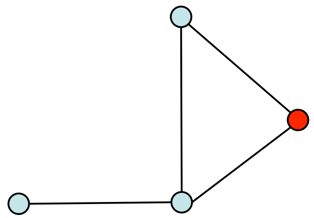
$$\dot{x}_1 = -(x_1 - u),\ \dot{x}_2 = -(x_2 - u)$$
$$x_1(0) = x_2(0) \ \Rightarrow \ x_1(t) = x_2(t)\ \forall u, t \geq 0$$



Not controllable!

Why? - Same reason!

$$\dot{x}_1 = -(x_1 - u) - (x_1 - x_2)$$
$$\dot{x}_2 = -(x_2 - u) - (x_2 - x_1)$$
$$x_1(0) = x_2(0) \ \Rightarrow \ x_1(t) = x_2(t)\ \forall u, t \geq 0$$

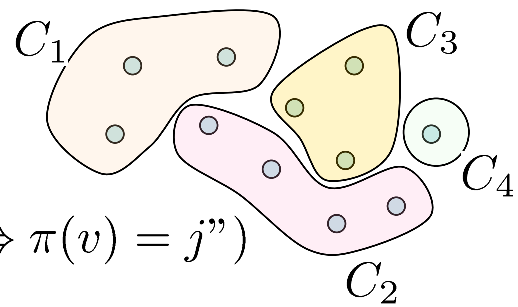


Controllable!

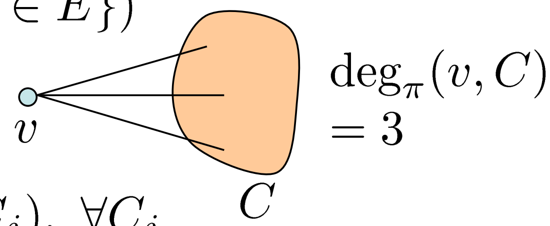
Why? - Somehow it seems like some kind of "symmetry" has been broken.

Symmetry? - External Equitable Partitions

- Given a graph
$$G = (V, E)$$

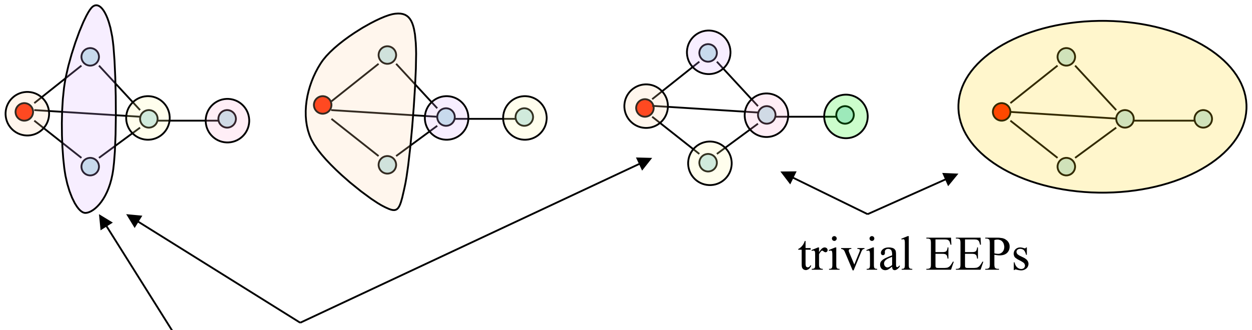


- Define a **partition** of the node set into cells
$$\pi : V \to \{1, \ldots, K\}, \quad (\text{"}v \in_\pi C_j \Leftrightarrow \pi(v) = j\text{"})$$
- Let the **node-to-cell degree** be given by
$$\deg_\pi(v, C) = card(\{v' \in_\pi C \mid (v, v') \in E\})$$



- The partition is an **equitable partition** if
$$\pi(v) = \pi(v') \;\Leftrightarrow\; \deg_\pi(v, C_j) = \deg_\pi(v', C_j), \; \forall C_j$$
- The partition is an **external equitable partition** if
$$\pi(v) = \pi(v') \;\Leftrightarrow\; \deg_\pi(v, C_j) = \deg_\pi(v', C_j), \; \forall C_j, \; j \neq \pi(v)$$
(it does not matter what edges are inside a cell)

External Equitable Partitions



- An EEP is **leader-invariant** (LEP) if each leader belongs to its own cell
- A LEP is **maximal** if no other LEP with fewer cells exists

Controllability?

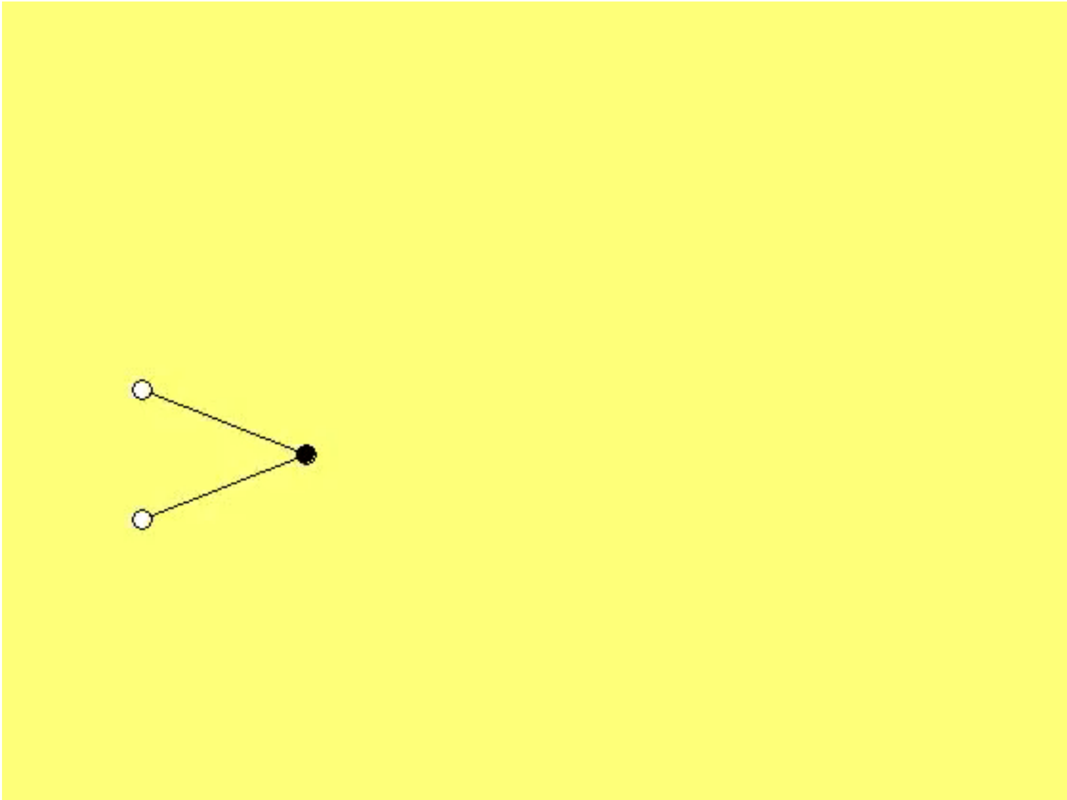
- From the leaders' vantage-point, nodes in the same cell "look" the same
- Let

$$\dot{x}_i = -\sum_{j \in N_i}(x_i - x_j),\ v_i \in V_F$$
$$\dot{x}_i = u_i,\ v_i \in V_L$$

- **Theorem** [7,8]: The uncontrollable part is asymptotically stable (if the graph is connected). It is moreover given (in part) by the difference between agents inside the same cell in the maximal LEP.

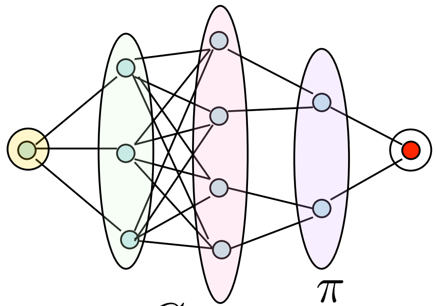
- **Corollary:** The system is completely controllable only if the only LEP is the trivial EEP

Uncontrollable Part

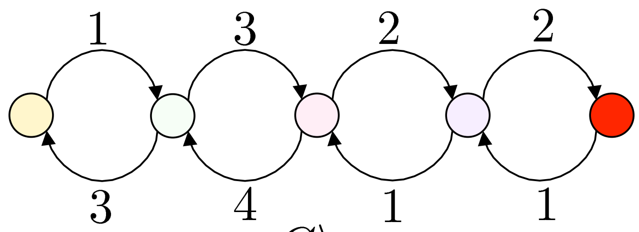


Quotient Graphs

- To understand the controllable subspace, we need the notion of a **quotient graph:**
  - Identify the vertices with the cells in the partition (maximal LEP)
  - Let the edges be weighted and directed in-between cells



G π



$G \backslash \pi$

- What is the dynamics over the quotient graph?

Quotient Graphs = Controllable Subspace

- Original system:

$$\Sigma_1 : \begin{cases} \dot{x}_i = -\sum_{j \in N_i} (x_i - x_j), \; v_i \in V_F \\ \dot{x}_i = u_i, \; v_i \in V_L \end{cases}$$

- Quotient graph dynamics:

$$\Sigma_2 : \begin{cases} \dot{\xi}_i = -\sum_{C_j \in N_{i,\pi}} \deg_\pi(C_j, C_i)(\xi_i - \xi_j), \; \pi(v) = i, \; v \in V_F \\ \dot{\xi}_i = u_i, \; \pi(v) = i, \; v \in V_L \end{cases}$$

- **Theorem** [8]**:**

$$\xi_i(0) = \frac{1}{|C_i|} \sum_{j \mid \pi(v_j) = i} x_j(0) \;\Rightarrow\; \xi_i(t) = \frac{1}{|C_i|} \sum_{j \mid \pi(v_j) = i} x_j(t)$$

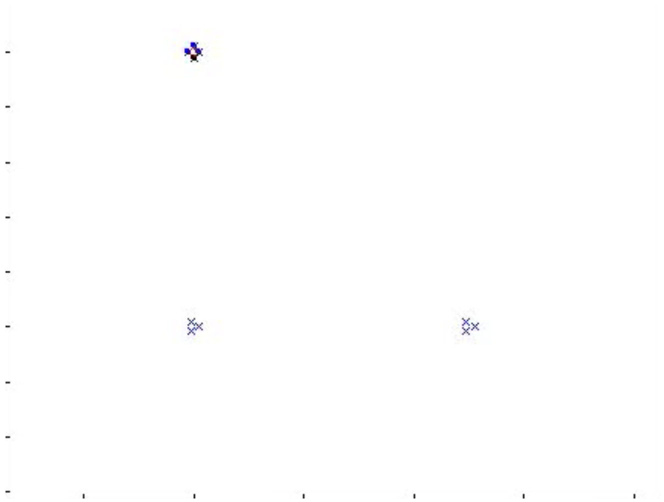
Graph-Based Controllability

- So what have we found?
  1. The system is completely controllable only if the only LEP is the trivial LEP
  2. The controllable subspace has a graph-theoretic interpretation in terms of the quotient graph of the maximal LEP
  3. The uncontrollable part decays asymptotically (all states become the same inside cells)
  4. Why bother with the full graph when all we have control over is the quotient graph? (= smaller system!)

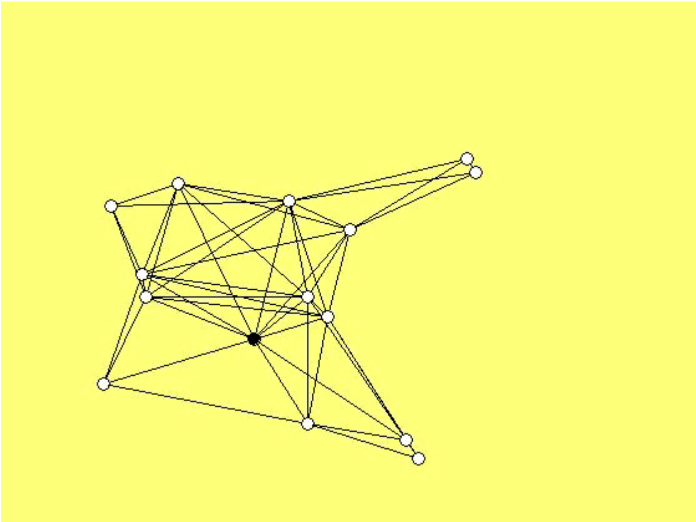
- **Now, let’s put it to use!**

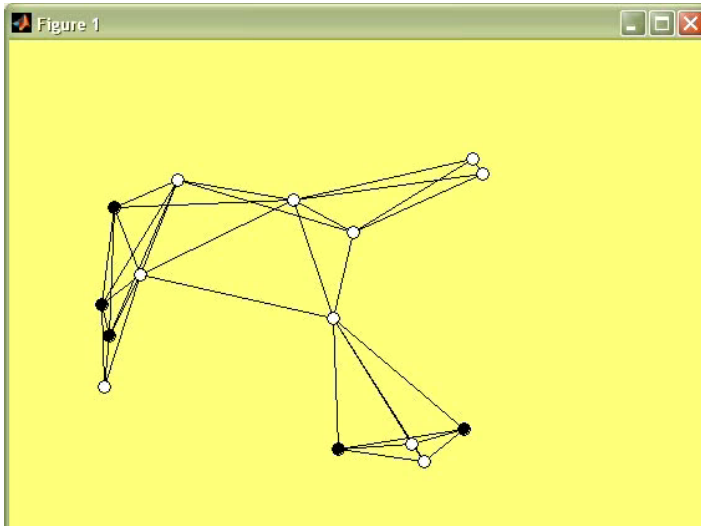
General Control Problems

- Controllability = We can solve general control problems for leader-based robot networks

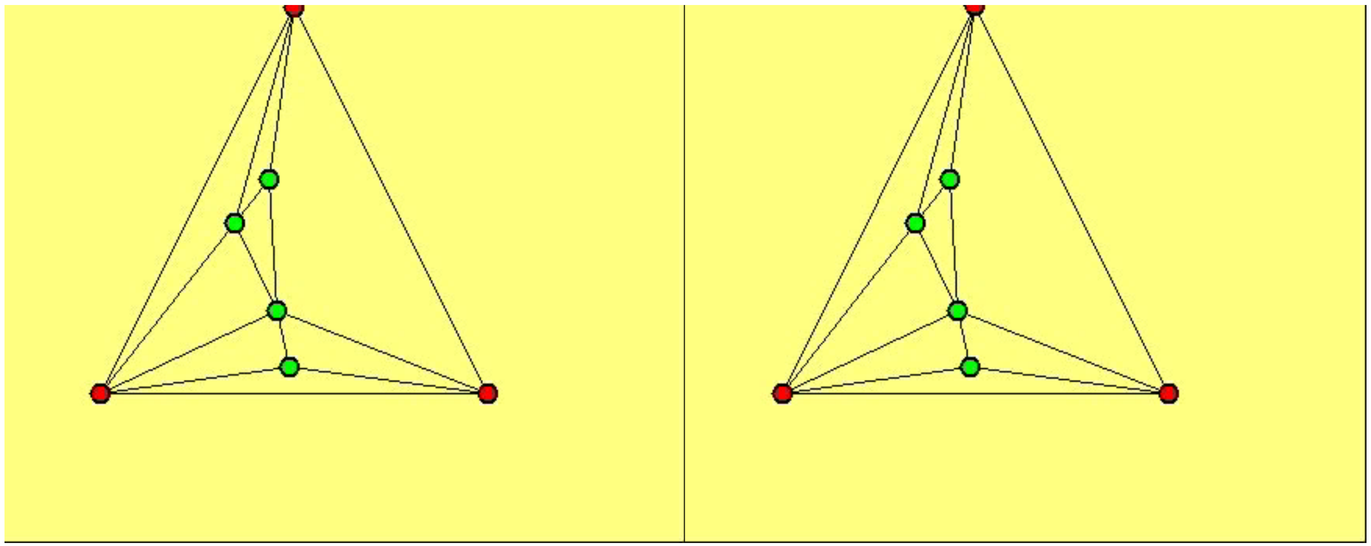


Stationary Leaders as Anchors



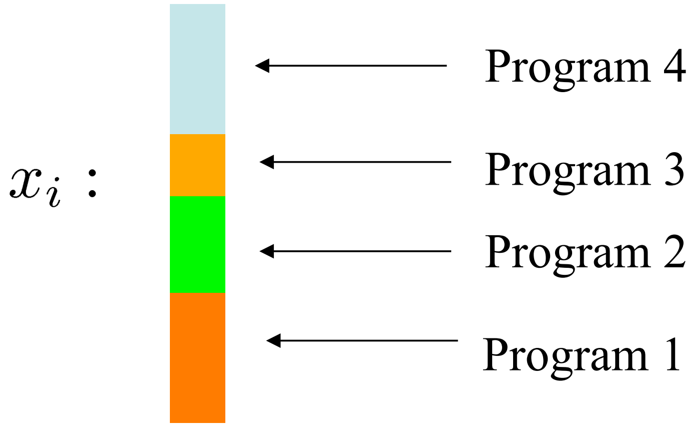


Containment Control



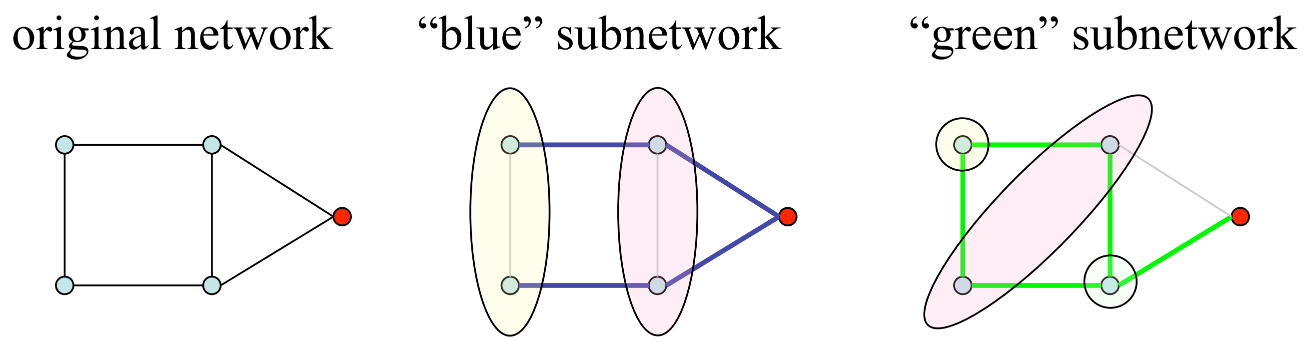
Epidemic Programming

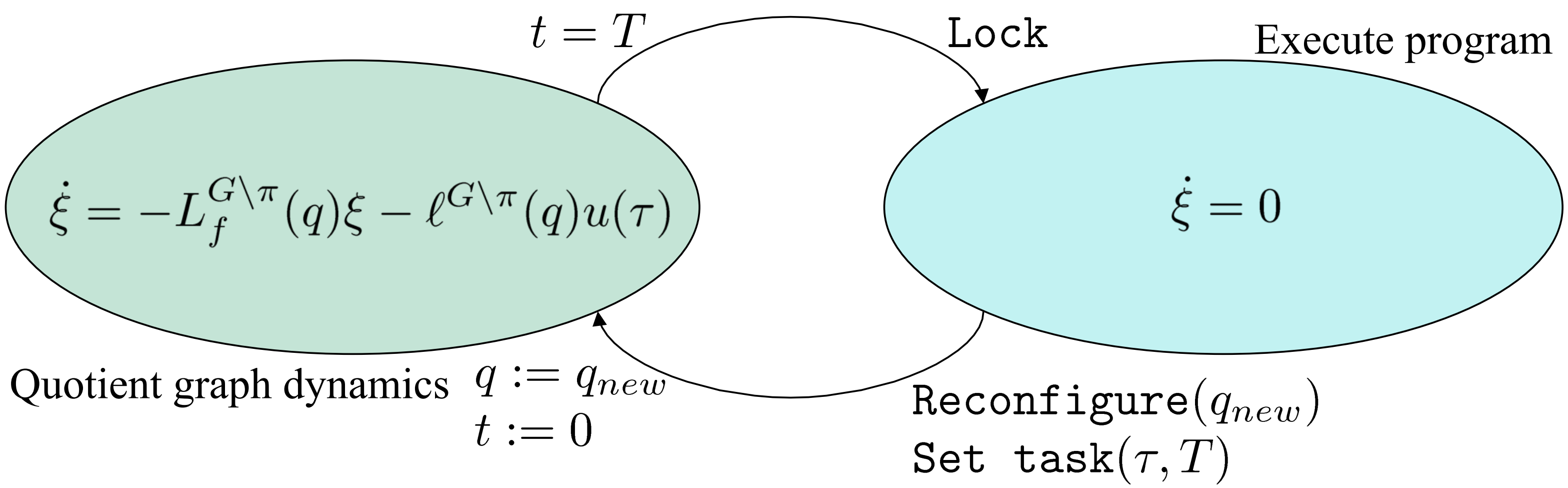
- Given a scalar state of each agent whose value determines what "program" the node should be running



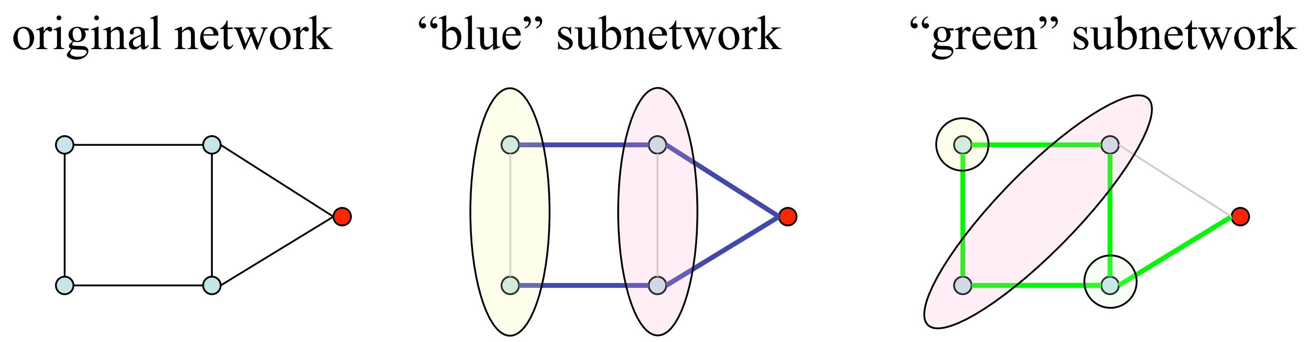
- By controlling this state, new tasks can be spread through the network
- But, we do not want to control individual nodes – rather we want to specify what each node "type" should be doing
- Idea: Produce sub-networks that give the desired LEPs and then control the system that way

Epidemic Programming



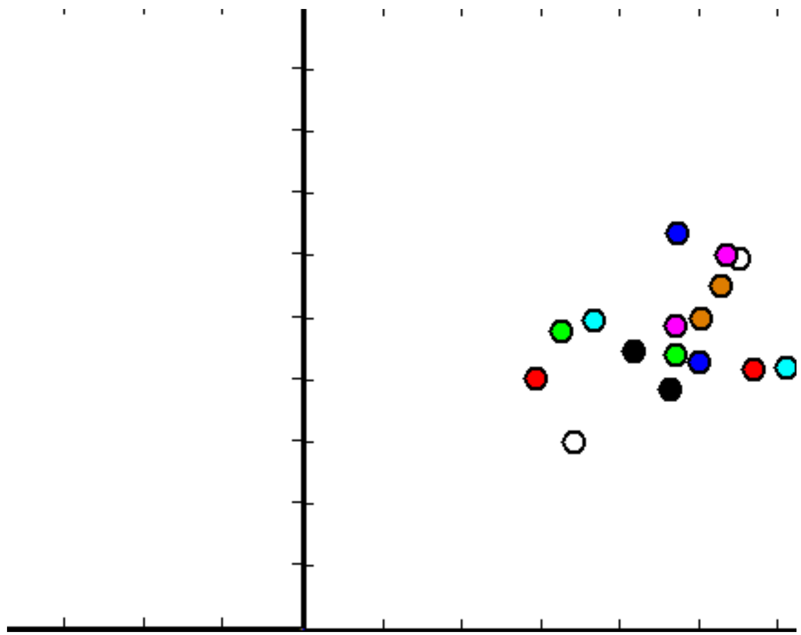


Epidemic Programming

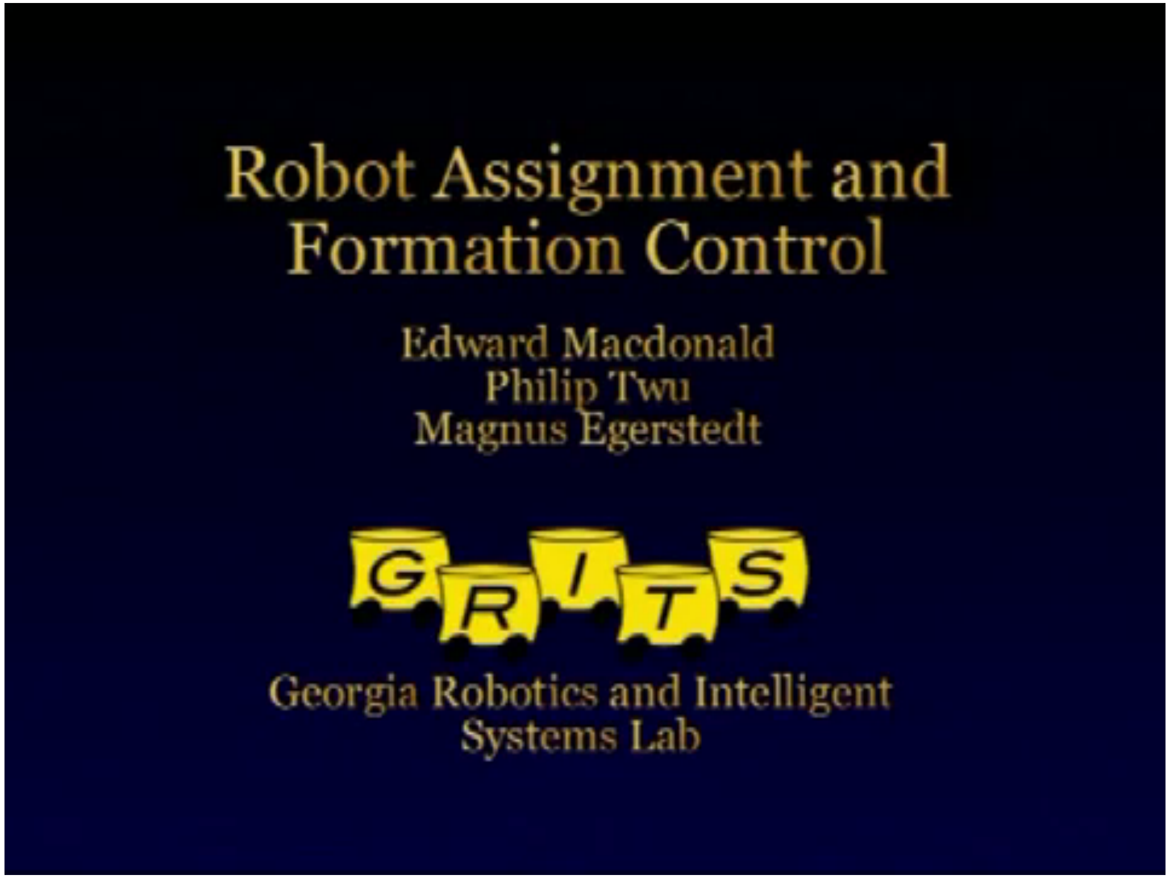


- Given a complete graph and a desired grouping of nodes into cells, produce a maximal LEP for exactly those cells using the fewest possible edges. (Answer is surprisingly enough not a combinatorial explosion…)

Epidemic Programming



Heterogeneous Networks



Summary III

- By introducing leader-nodes, the network can be “reprogrammed” to perform multiple tasks such as move between different spatial domains
- Controllability based on graph-theoretic properties was introduced through external equitable partitions