SESSION 4
SENSOR NETWORKS

Introduction

- Sensor networks are becoming an important component in cyber-physical systems:
  - smart buildings
  - unmanned reconnaissance




- Limited power capacity requires algorithms that can maintain area coverage and limit power consumption.

Node Models

- Consider a network of N sensors, with the following characteristics:

$p_i \in \Re^2$ ← position

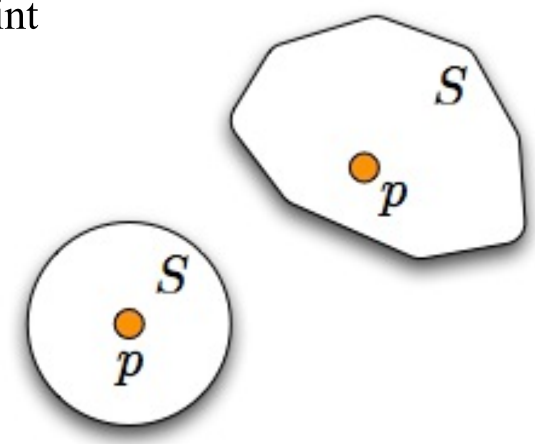
$\eta_i \in \Re_+$ ← power level

$S_i \subset \Re^2$ ← sensor footprint

- For example – standard disk model

$$S_i = \{x \in \Re^2 \mid \|x - p_i\| \leq \Delta\}$$

- Question: What is the connection between power level and performance?



Node Models

- A sensor can either be awake or asleep

$$\sigma = \begin{cases} 1 \\ 0 \end{cases}$$

sensor on

sensor off

- Power usage

$$\dot{\eta} = f_{pow}(\eta, \sigma), \quad \sigma = 0 \;\Rightarrow\; \dot{\eta} = 0$$

- Sensor footprint

$$S = S(p, \eta, \sigma), \quad \sigma = 0 \;\Rightarrow\; S = 0$$

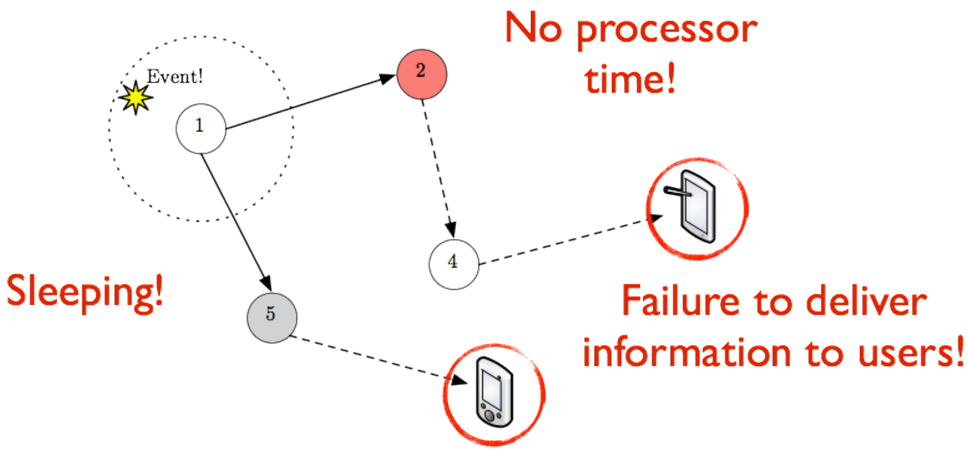
- Mobility

$$\dot{p} = f_{mob}(p, \eta, u)$$

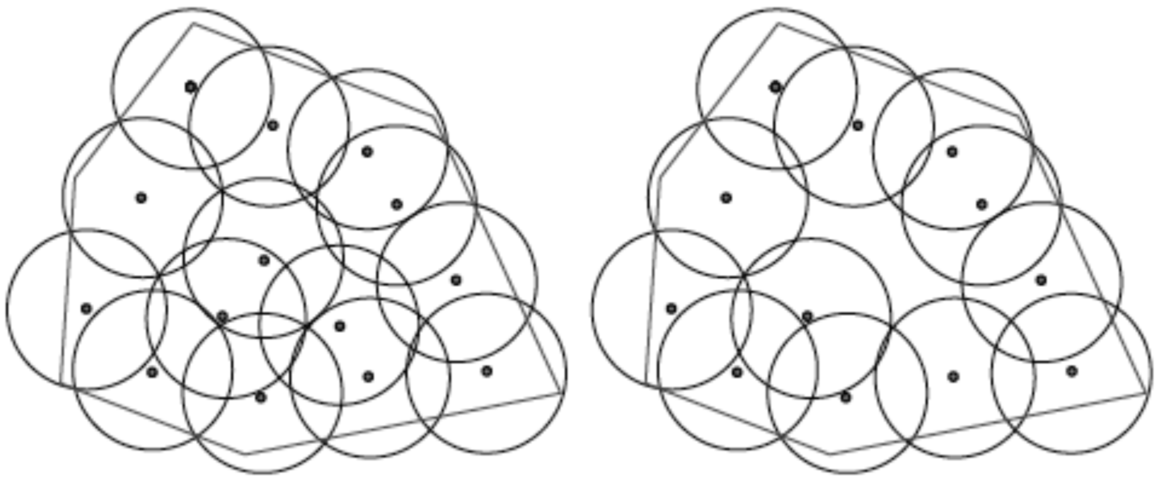
Node-level control variables

Node Models

- The available power levels affect the performance of the sensor nodes
- Sensor footprint – RF or radar-based sensors
  - Decreasing power levels leads to shrinking footprints
- Frame rates – vision based sensors
- Latency issues across the communications network



Coverage Problems



Coverage Problems

- Given a domain M. ***Complete coverage*** is achieved if

$$M \subseteq \bigcup_{i=1}^{N} S_i$$

- Areas are easier to manipulate than sets, and ***effective area coverage*** is achieved if

$$m \leq \left| \bigcup_{i=1}^{N} S_i \right|$$

$$G_{cov}(S) \geq 0$$

- Instead one can see whether or not events are detected with ***sufficient even detection probability***

$$\mu \leq \text{prob}\left(\text{event} \in \bigcup_{i=1}^{N} S_i \right)$$

Coverage/Life-Time Problems

- Now we can formulate the general life-time problem as

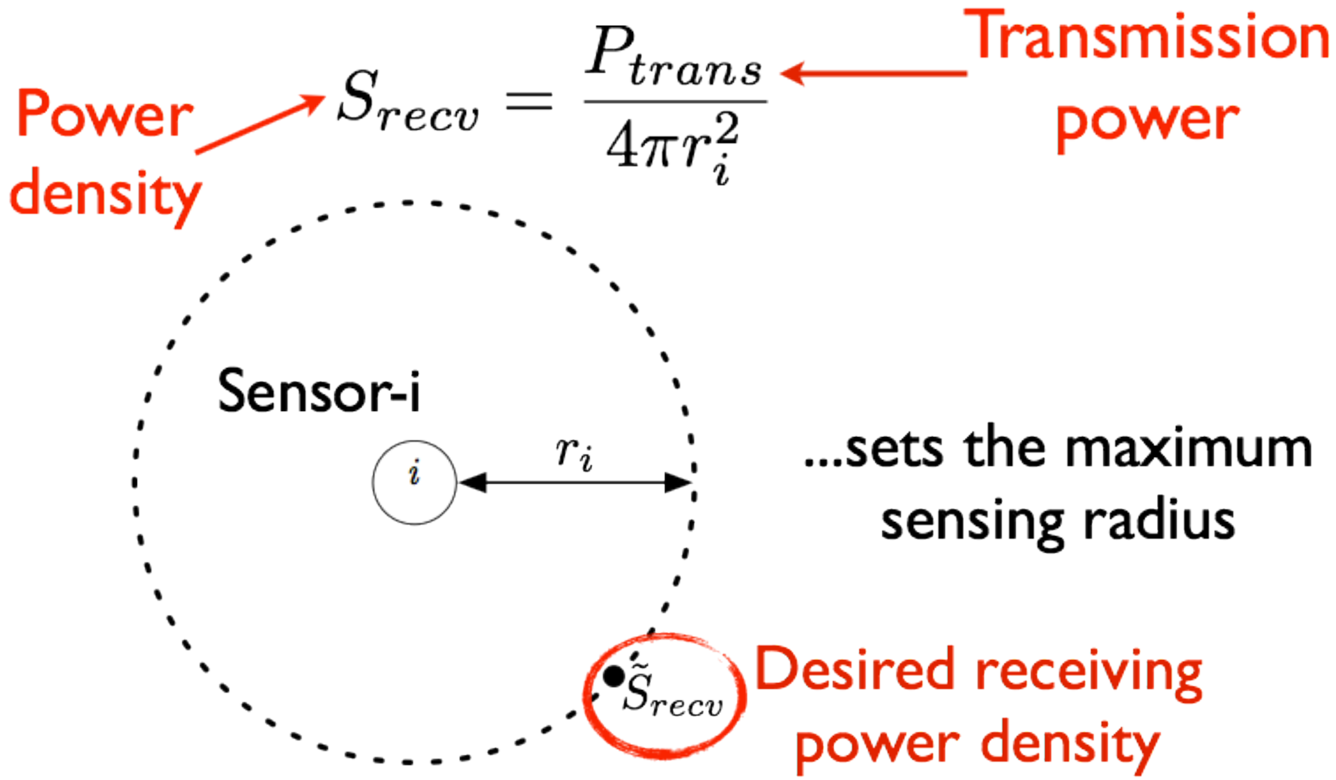
$$\max\ T \text{ such that } G_{cov}(S(T)) \geq 0,\ \forall t \leq T$$

- We will address this for some versions of the problem
  - Node-based, deterministic
  - Ensemble-based, stochastic

Radial Sensor Model

- Assume an isotropic RF transmission model for each sensor:

$$S_{recv} = \frac{P_{trans}}{4\pi r_i^2}$$



Radial Sensor Model

- Area covered by sensor is given by:

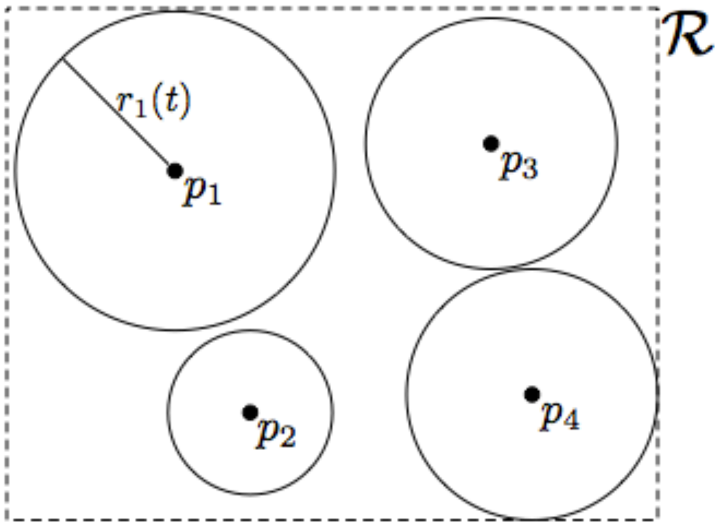
$$\pi r_i(t)^2 = \frac{P_{trans}}{4\tilde{S}_{recv}}$$

- But, sensor-i's transmitted power depends on its current power level:

$$P_{trans} = \sigma_i \eta_i$$

- Footprint:

$$|S(\eta_i, \sigma_i)| = \pi r_i^2(t) = \frac{\sigma_i \eta_i}{4\tilde{S}_{recv}}$$



Problem Formulation

- Our goal is effective area coverage, i.e.,

$$m \leq \left| \bigcup_{i=1}^{N} S_i \right|$$

- Assume sensor footprints do not intersect, then:

$$\left| \bigcup_{i=1}^{N} S_i \right| = \sum_{i=1}^{N} |S_i| \overset{\text{(almost)}}{=} \sum_{i=1}^{n} \sigma_i \eta_i$$

- Coverage constraint:

$$G_{cov}(S(t)) = \sum_{i=1}^{N} \sigma_i(t)\eta_i(t) - m \geq 0$$

Optimal Control

- Let

$$x = [\eta_1, \ldots, \eta_N]^T, \quad u = \mathrm{diag}(\sigma_1, \ldots, \sigma_N)$$

- Aggregate dynamics

$$\dot{x}(t) = -\gamma u(t) x(t)$$

- Problem: Find gain signals that solve

$$\min_u J(u, x, t) = \int_{t_0}^{T} \frac{1}{2} \Big(\big(u^T(t)x(t) - M\big)^2 + u^T(t) R u(t) \Big) \, dt$$

Optimal Control

Hamiltonian:

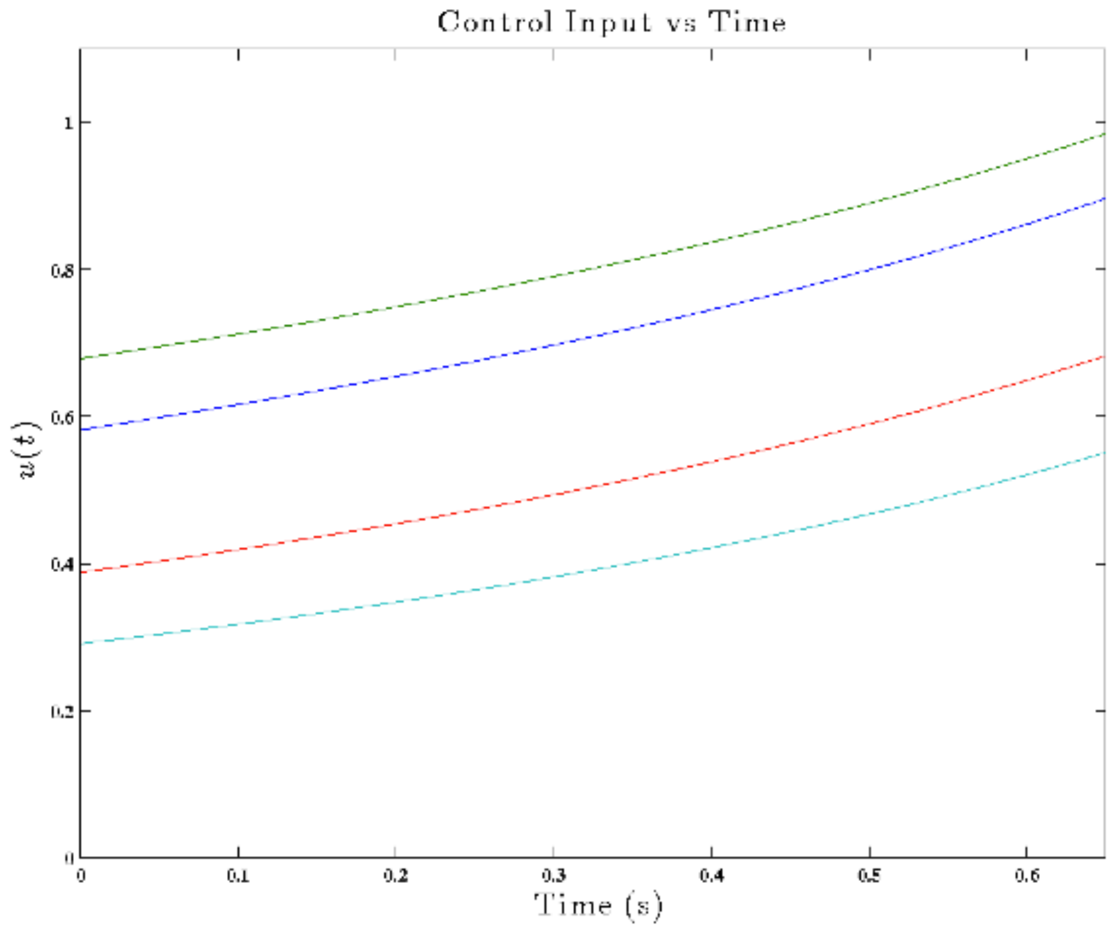
$$H(u,x,t) = -u^T(t)\Lambda(t)x(t) + \frac{1}{2}\left(u(t)^T x(t) - M\right)^2 + \frac{1}{2}u(t)^T R u(t)$$

Where $\Lambda(t) = diag(\lambda_i(t))$ represents the co-states satisfying the backward differential equation:

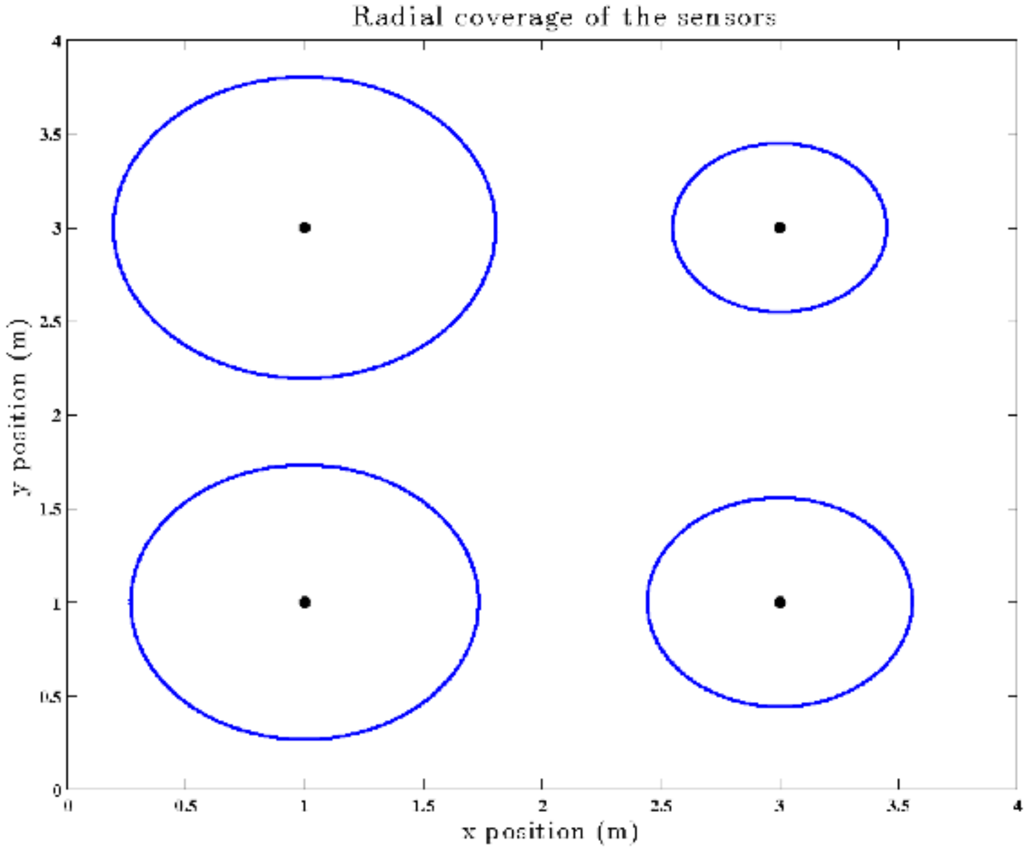
$$\dot{\lambda}(t) = \Lambda(t)u(t) - \left(u(t)^T x(t) - M\right)u(t), \lambda(T) = 0$$

Optimal gain signals:

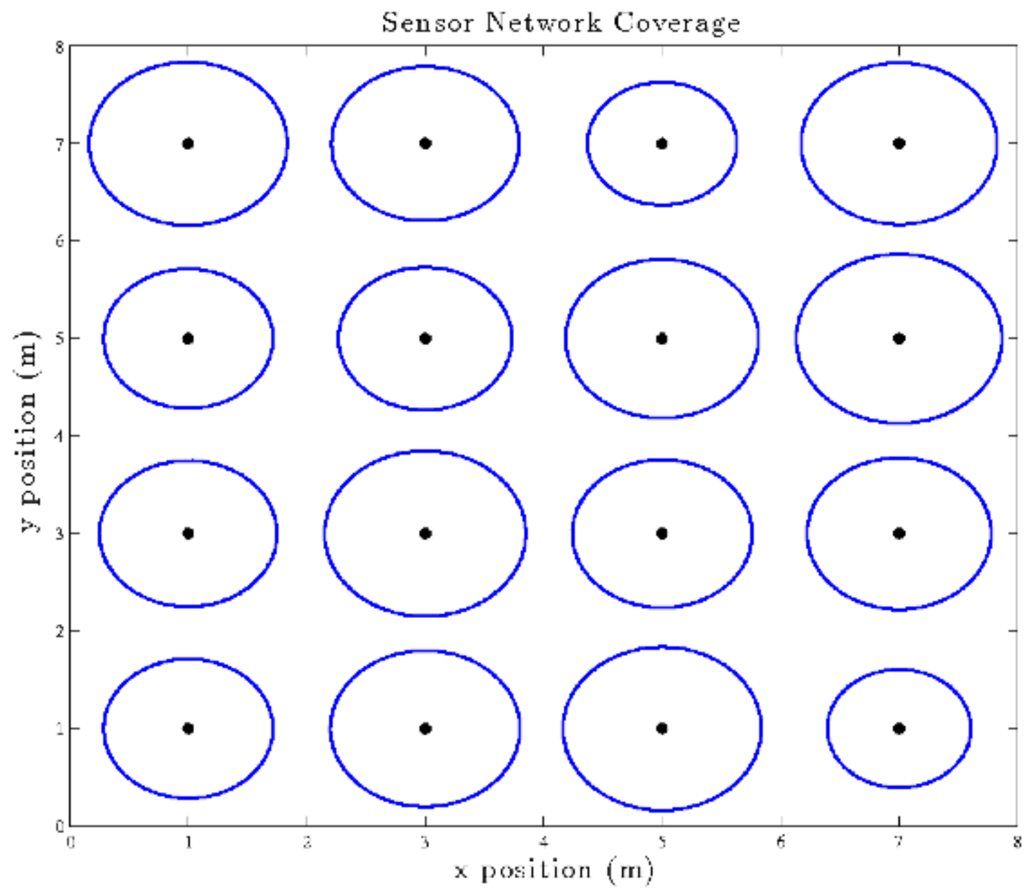
$$u(t) = \left(x(t)x^T(t) + R\right)^{-1}\left(\Lambda(t) + MI\right)x(t)$$



Control Input vs Time
1
0.8
0.6
0.4
0.2
0
$u(t)$
0
0.1
0.2
0.3
0.4
0.5
0.6
Time (s)



Radial coverage of the sensors
y position (m)
x position (m)
0
0.5
1
1.5
2
2.5
3
3.5
4



Sensor Network Coverage
y position (m)
x position (m)
0
1
2
3
4
5
6
7
8

Issues

- Maybe not the right problem:
  - No on/off (relaxation)
  - No life-time maximization
- What we do know about the "right" problem
  - Only switch exactly when the minimum level is reached
  - Knapsack++
- Maybe we can do better if we allow for randomness in the model?

The Setup

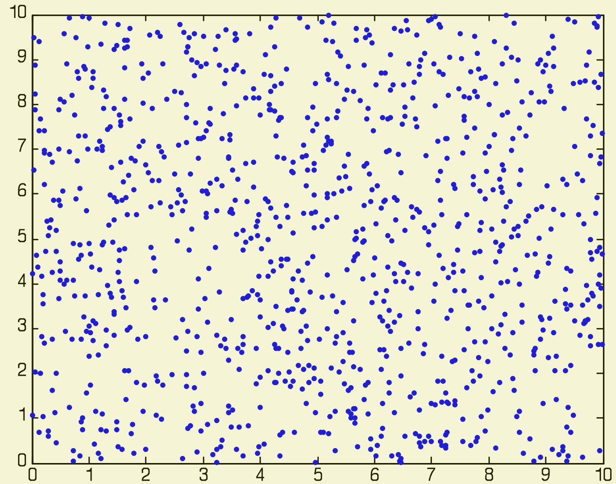
- Given a decaying sensor network we want to find a scheduling scheme that maintains a desired network performance throughout the lifetime of the network.
- The desired network performance is the minimum satisfactory probability of an event being detected.
- Lifetime of the sensor network is the maximal time beyond which the desired network performance cannot be achieved.
- We assume that the sensor nodes are “dropped” over an area.

Spatial Poisson Processes

- We assume that the sensor nodes are dropped according to a spatial Poisson point process:

i. The number of points in any subset X of *D*, n(X), are Poisson distributed with intensity λ||X||, where λ is the intensity per unit area.

ii. The number of points in any finite number of disjoint subsets of *D* are independent random variables.



$$P(n \text{ sensors in area } A) = \frac{(\lambda A)^n e^{-\lambda A}}{n!}$$

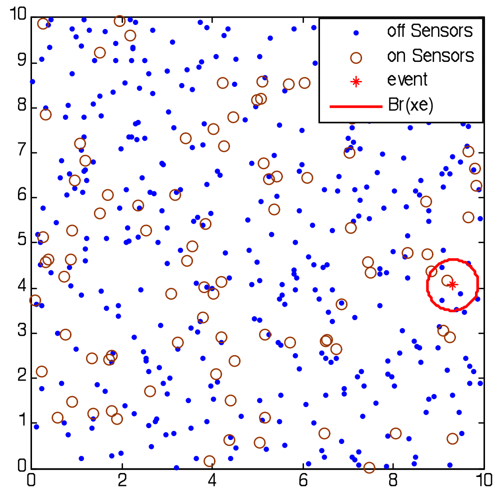
System Model

- All sensors are identical i.e., they have same
  - Initial power and power decay rate
  - Sensing capabilities
- All sensors have circular footprint

$$S_i = B_r(p_i)$$

  - An event at location x_e is detected if

$$x_e \in B_r(p_i)$$



- To conserve power, sensors are switched between on state and off state
  - Power is consumed only when a sensor is on: $\dot{\eta}_i = -\gamma q_i \eta_i$

Prob that sensor is on at time t

Event Detection Probability

- Consider a non-persistent event
  - An event is non-persistent if it does not leave a mark in the environment and can only be detected when it occurs.
- Theorems:
  - Probability of an event going undetected by a non-decaying sensor network is

$$P_u = e^{-\lambda \pi r^2 q}$$

  - Probability of an event going undetected by a decaying sensor network is

$$P_u = e^{-\lambda c e^{-\gamma \int_0^t q(s) ds} q(t)}$$

$$r^2(t) \propto \eta(t)$$

$$A(t) = \pi r(t)^2 = c e^{-\gamma \int_0^t q(s) ds}$$

Controlling Duty Cycles

- We need a controller of the form

$$\dot{q}(t) = u(t)$$

to maintain a constant P_d (as long as possible)

- Controller:

$$q(0) = \frac{\ln\left(\frac{1}{1-P_d}\right)}{\lambda c}$$

$$u(t) = \gamma q(t)^2$$

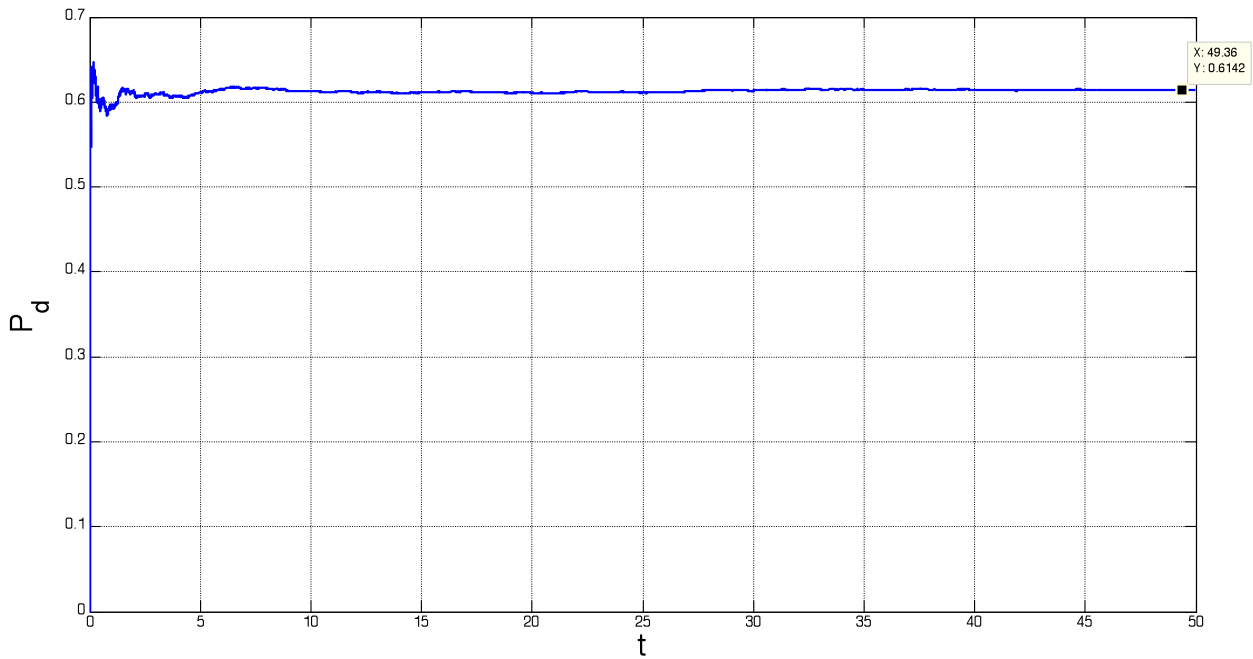
- Life time:

$$T = \frac{1}{\gamma}\left(\frac{\lambda c}{\ln\left(\frac{1}{1-P_d}\right)} - 1\right)$$

Simulation Results

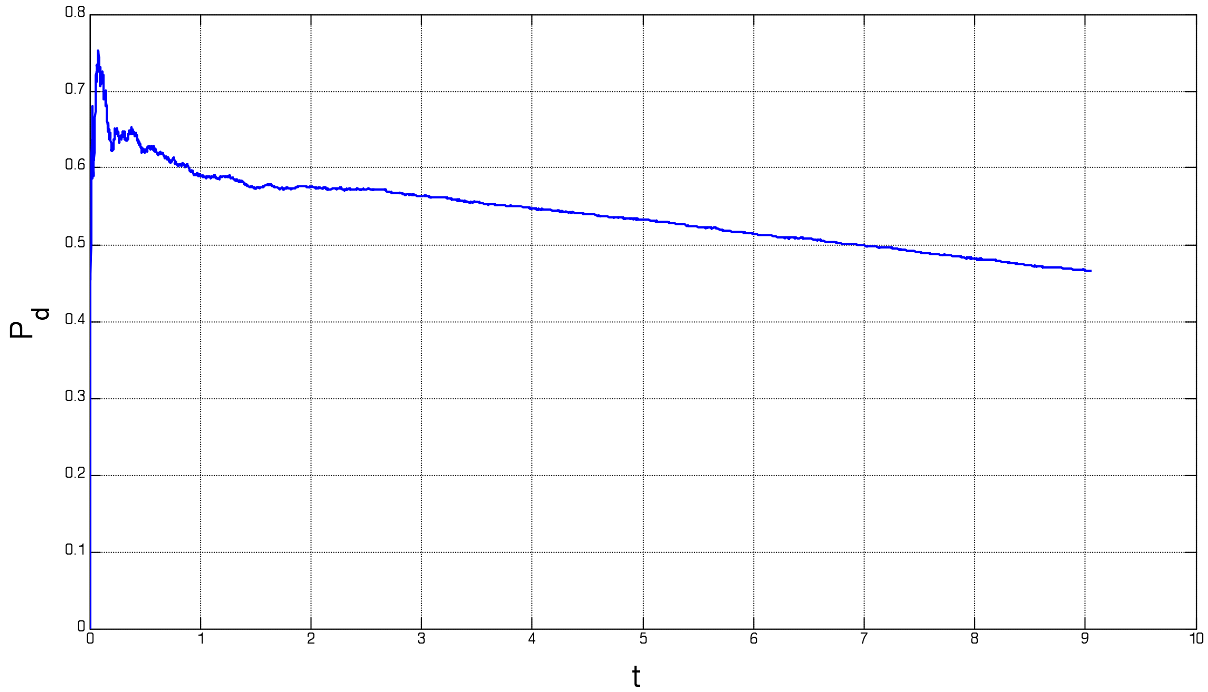
- A Monte – Carlo simulation of the network is performed
- In a (10 x 10) unit rectangular region sensors are deployed according to a spatial stationary Poisson point process with intensity $\lambda = 10$.
- Different scenarios (non – decaying network, decaying network, decaying network with scheduling scheme) are simulated with the following parameters
  - λ (intensity per unit area) = 10
  - γ (power decay rate) = 1
  - P_d (desired probability of event detection) = 0.63

Non-Decaying Footprints



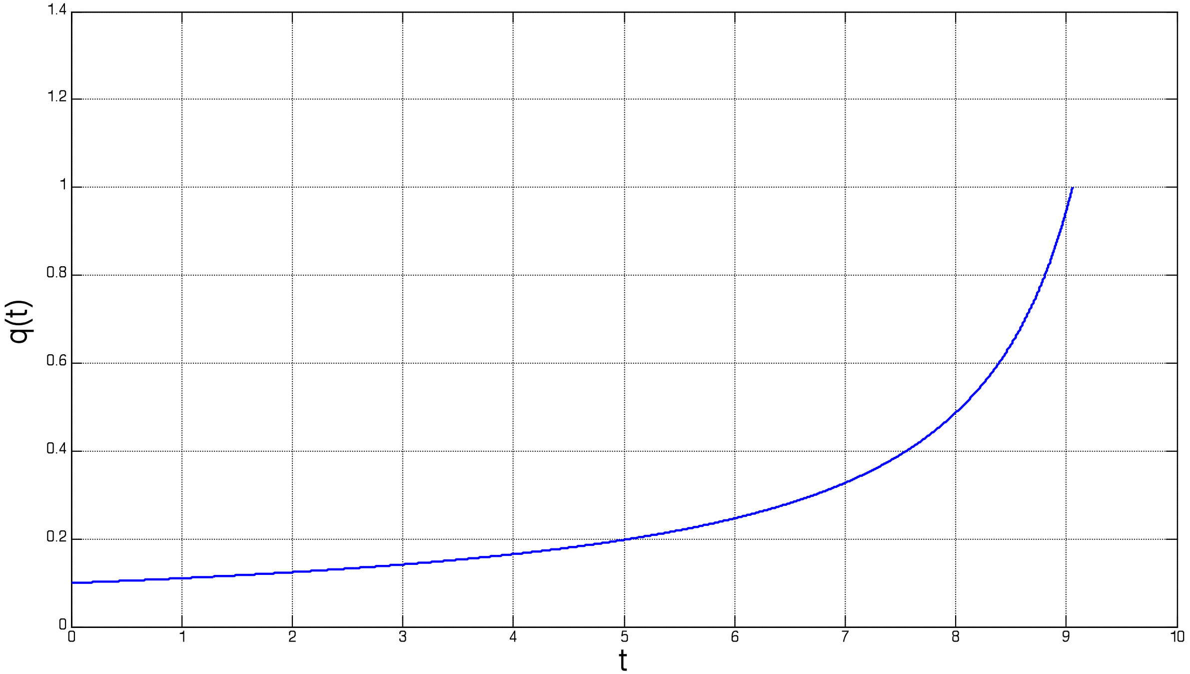
Event detection probability *Pd* vs time *t* for non-decaying networks with q = 0.1.

Decaying Footprints Without Feedback



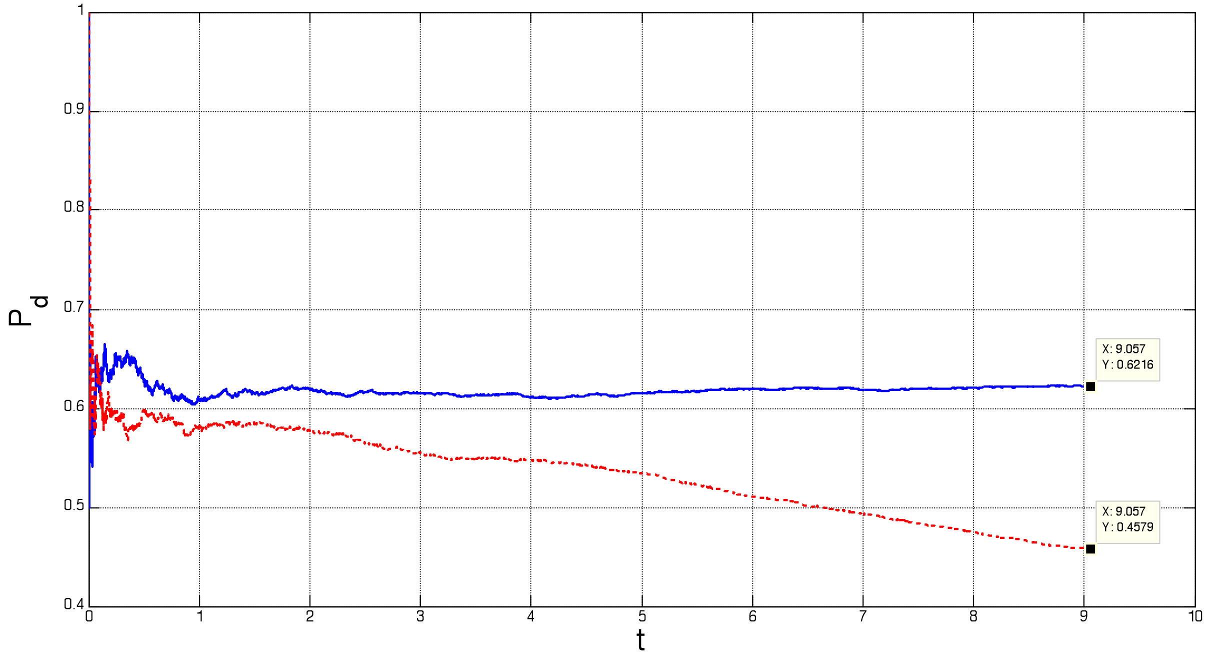
Event detection probability P_d vs time t for decaying networks with q = 0.1 and decay rate $\gamma = 1$

Decaying Footprints With Feedback

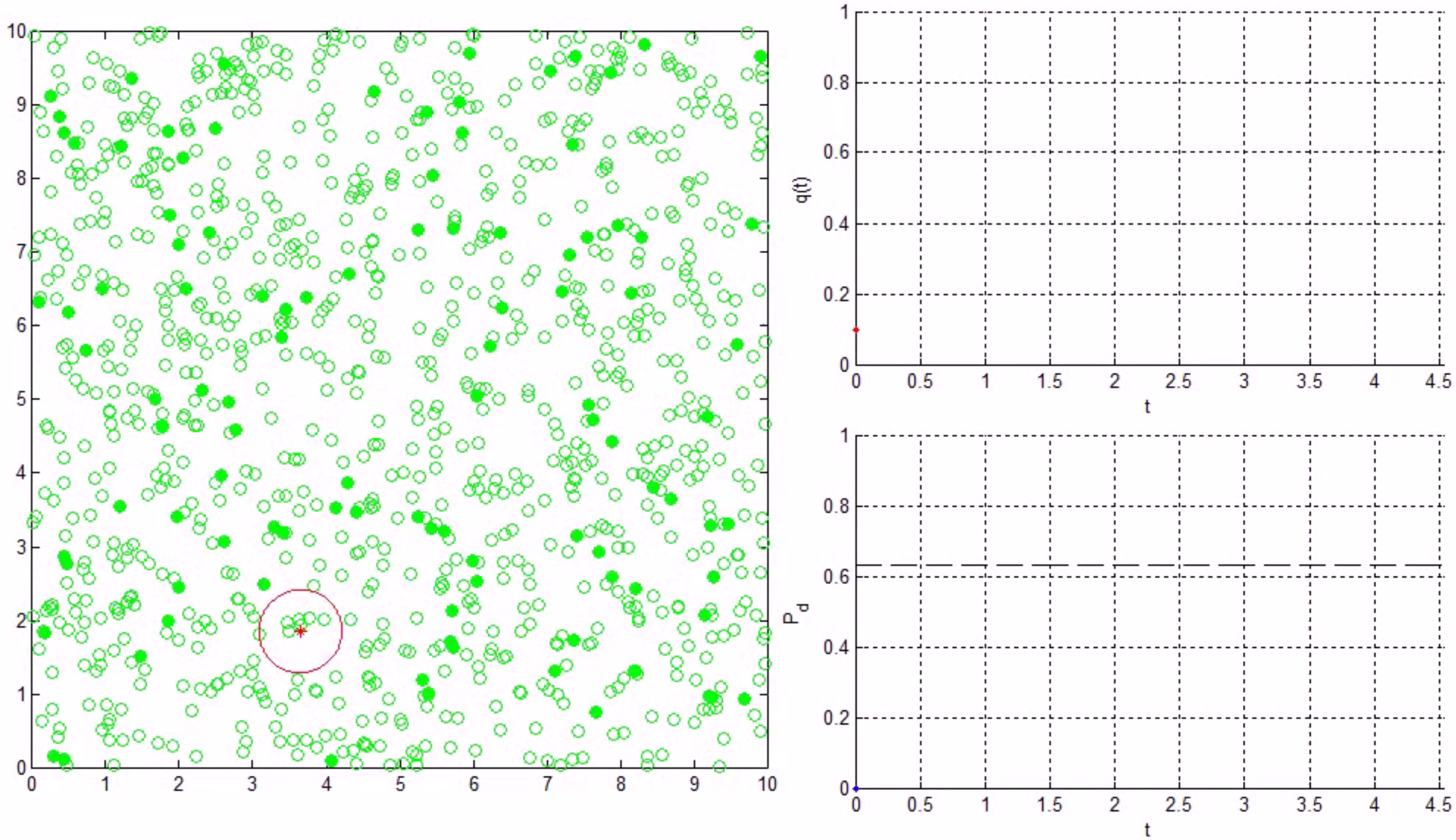


Probability of sensor being *on q vs time t*

Decaying Footprints With Feedback



Event detection probability P_d vs time t for decaying networks with given $P_d = 0.63$; with scheduling scheme (solid line) and without scheduling scheme (dashed line)


q(t)
t
P_d
t

Issues

- We may still not have the right problem:
  - No on/off cost
  - No consideration of the decreasing communications capabilities
- What we do know about the hard problem
  - Rendezvous with shrinking footprints while maintaining connectivity?
- **Big question**: Mobility vs. Sensing vs. Communications vs. Computation???

Summary IV

- By introducing power considerations into the formulation of the coverage problem, a new set of issues arise
- Life-time problems
- Shrinking footprints
- Ensemble vs. node-level design
- **Big question**: Mobility vs. Sensing vs. Communications vs. Computation???

Conclusions

- The graph is a useful and natural abstraction of the interactions in networked control systems
- By introducing leader-nodes, the network can be "reprogrammed" to perform multiple tasks such as move between different spatial domains
- Controllability based on graph-theoretic properties was introduced through external equitable partitions
- Life-time problems in sensor networks

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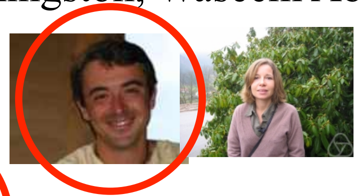

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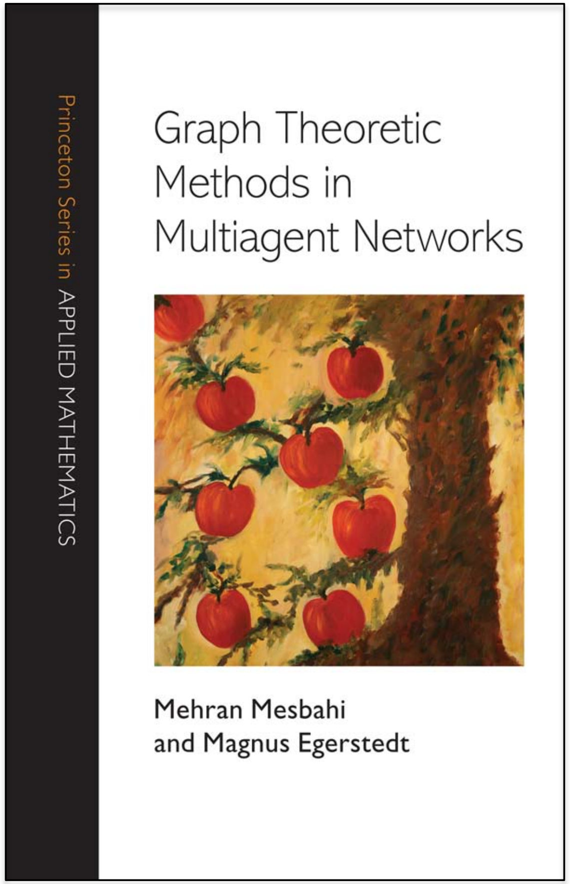
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