Pole Placement Design of Power Control Algorithms

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Abstract

Time delays reduces the performance of any controlled system. If neglected in the design phase, the system may even become unstable when using the designed controller. Several power control strategies have been proposed in order to improve the capacity of cellular radio systems, but time delays are usually neglected. Here, it is shown that the problems can be handled by considering the time delays in the design phase in order to choose the appropriate parameter values. Most popular algorithms can be seen as special cases of an integrating controller. This structure is extended first to a PI-controller and then further on to a general linear controller of higher orders. Corresponding design procedures are outlined based on techniques, such as pole placement, from the field of automatic control. The PI-controller is a very appealing choice of structure, with better performance compared to an I-controller and less complex than a higher order controller. The benefits are further illuminated by network simulations.

Keywords: Cellular radio systems; Power Control Algorithms; Pole placement; Local loop design; Linear controllers

1 Introduction

For proper operation of a high-capacity cellular radio system, power control is an essential feature. Among others, there are three important aspects to consider.

- Extracting relevant information from the available measurements.
- Design a linear power control algorithm and tune the parameter for optimized performance.
- Incorporate nonlinear components to handle constraints and priorities.

The second issue is dealt with here, while the others are discussed in [12, 13]. Several transmitter power control algorithms have been proposed to date. Most schemes strive to balance the *carrier-to-interference* ratios (C/I) on each channel such that every mobile or base station achieve the same C/I [17].

To avoid extensive control signaling in the network, it is desirable to use distributed algorithms, where the transmitter powers are locally controlled based on local measurements (e.g. C/I). Such distributed algorithms have previously been studied in [8, 7, 1, 16, 14, 2, 15].

These algorithms perform well in rather ideal cases, but in real systems there are a number of effects that hamper the performance as discussed in [5, 6, 10]. Among the most troublesome are the effects from time delays in the system. The stability analysis in [5, 11] reveals that the Distributed Power Control (DPC) algorithm [8, 7], which works fine under ideal circumstances, yields an unstable system when subject to a small time delay.

When the time delay is identified, the information can be considered not only in stability analysis, but also in the design phase in order to reduce its effects as much as possible. Most popular algorithms can be seen as special cases of an integrating controller [5], which is generalized into a Proportional Integrating (PI) controller. Then, the issue of performance measures and specifications are discussed. Techniques from the field of automatic control, such as pole placement design will be reviewed and applied. As always, the solution can be seen as a tradeoff between ability to track fast variations and to attenuate disturbances. Some illuminating simulations and conclusions conclude the paper.

2 System Model

Signal gains and power levels can be expressed using either logarithmic (e.g. *dB* or *dBm*) or linear scale. To avoid confusion we will employ the convention of indicating linearly scaled values with a bar. Thus $\bar{g}_{ij}$ is a value in linear scale and g_{ij} the corresponding value in logarithmic scale.

Assume that the m mobile stations on a specific radio channel are transmitting using the powers $p_i(t)$, $i = 1, \ldots, m$. The signal between mobile station

i and base station j is attenuated by the signal gain $g_{ij}(t)$ (< 0). Thus the corresponding connected base station will experience a desired carrier signal $C_i(t) = p_i(t) + g_{ii}(t)$ and an interference plus noise $I_i(t)$ (values in logarithmic scale). The *carrier-to-interference* ratio at base station i is defined by

$$\gamma_i(t) = p_i(t) + g_{ii}(t) - I_i(t). \tag{1}$$

We will assume that the *Quality of Service* (QoS) is depending only on the C/I, and is acceptable iff

$$\gamma_i(t) \geq \gamma^*, i = 1, \ldots, m. \tag{2}$$

The measured (or estimated) C/I:s, denoted by $\hat{\gamma}_i$, may be inaccurate or corrupted by noise. Therefore various filters $F(q)$ can be used to obtain smooth estimates. As concluded in [11], an exponential forgetting (EF) filter is an interesting choice. It is defined by the following recursion

$$\gamma_i(t+1) = \lambda\gamma_i(t) + (1-\lambda)\hat{\gamma}_i(t+1)\ , \ \ 0 \leq \lambda < 1,$$

or using the delay operator q ($q^{-n}\gamma(t) = \gamma(t-n)$)

$$\gamma_i(t) = \frac{(1-\lambda)q}{q-\lambda}\hat{\gamma}_i(t) = F_{EF}(q)\hat{\gamma}_i(t). \tag{3}$$

Primarily, there are two motives for considering the filter. On the one hand, it can be introduced as a device to reduce the noise. The parameter λ is then a design variable. On the other hand, it can be seen as a simple model of an estimator, when studying its effects on the dynamics of the power control loop. Then λ is a fixed value, which should be identified by experiments or modeling.

The output powers are considered to be unconstrained. Appropriate modifications when considering constrained output powers are discussed in [2, 9, 13].

A general distributed power control algorithm $R(q)$ and its surroundings can be depicted as in Figure 1, and this loop will be referred to as the *local loop*. Since the focus is on a single connection, the index i will be dropped for clarity. The power outputs and measurements are delayed by n_p and n_m samples respectively. In this work, the typical situation $n_p = n_m = 1$ will be studied. The measurement noise or estimation error $v(t)$ is a nuisance signal . Moreover, the objective is to track a target signal (possibly constant) $\gamma_{tgt}(t)$. From (2) it is natural to choose $\gamma_{tgt}(t) \geq \gamma^*$.

3 Controller Structures

3.1 I-controller

Many of the algorithms discussed in previous work, e.g. the DPC algorithm [8, 7], the Constant Received Power (CRP) algorithm [2] and the algorithms proposed in [16, 14, 15], can be shown [5] to be special cases of the integrating

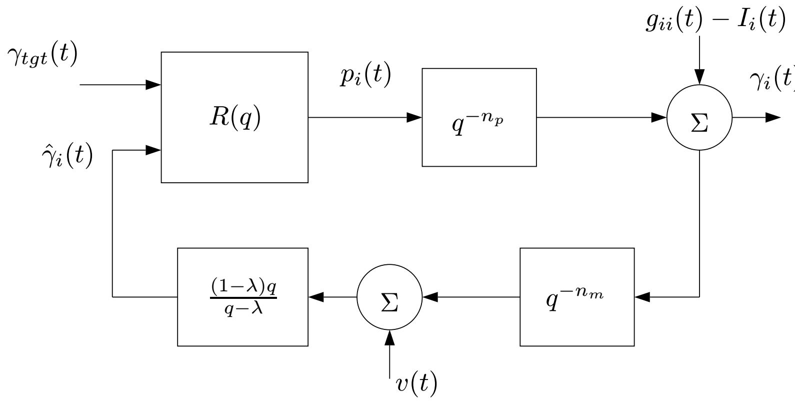

Figure 1: The local loop when employing the general power control algorithm $R(q)$ and the smoothing filter $F_{EF}(q) = \frac{(1-\lambda)q}{q-\lambda}$.

controller

$$p(t+1) = p(t) + K_i T_s \left(\gamma_{tgt}(t) - \hat{\gamma}(t)\right), \tag{4}$$

where T_s is the sampling interval and K_i is a design parameter. Define the error $e(t) = \gamma_{tgt}(t) - \hat{\gamma}(t)$ and using the delay operator, the I-controller can be expressed as

$$p(t) = \frac{K_i T_s q}{q-1} e(t) = R_I(q) e(t), \tag{5}$$

where an extra "q" is added to make the controller delay-less, and let n_m and n_p capture the delays.

3.2 PID-controller

A natural extension of (4) is the Proportional Integrating Differentiating (PID) [3] controller often considered in process applications. It can be described by

$$\begin{aligned} e(t) &= \left(\gamma_{tgt}(t) - \hat{\gamma}(t)\right) \\ x(t+1) &= x(t) + K_i T_s e(t) \\ p(t) &= K_p e(t) + x(t+1) + \frac{K_d}{T_s}\left(e(t) - e(t-1)\right), \end{aligned}$$

where $x(t)$ is the integration state, and K_p, K_i and K_d are parameters. It is common to omit the differentiation ($K_d = 0$) when the measurements are noisy. An alternative is to apply approximated differentiation [3]. In this paper, differentiation will be omitted. Using the delay operator, we obtain

$$p(t) = \frac{(K_p + K_i T_s)q - K_p}{q-1} e(t) = R_{PI}(q) e(t). \tag{6}$$

3.3 General Linear Controller

A more general linear controller is given by

$$p(t) = \frac{B_{ff}(q)}{A_{ff}(q)}\gamma_{tgt}(t) - \frac{B_r(q)}{A_r(q)}\hat{\gamma}(t), \tag{7}$$

where A_{ff}, B_{ff}, A_r and B_r are polynomials in the delay operator. By introducing the following notation, this general control loop can be depicted as in Figure 2

$$\frac{B(q)}{A(q)} = \frac{1}{q}, \frac{B_F(q)}{A_F(q)} = \frac{(1-\lambda)}{q-\lambda}$$

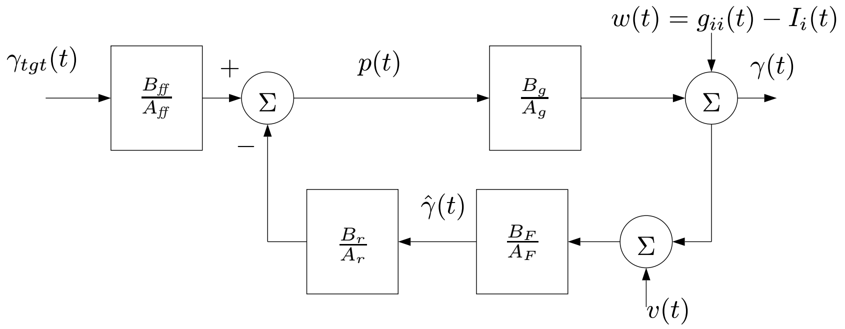

Figure 2: The general power control problem in local loop.

4 Properties of Linear Systems

The presentation here will be rather compact. For further details, see [4]. Consider the continuous-time system

$$y(t) = G_c(p)u(t) = \frac{B_c(p)}{A_c(p)}u(t),$$

where p is the differentiation operator $pu(t) = \dot{u}(t)$. The response to changes in $u(t)$ are described by the locations of the poles s_j of $G_c(p)$, i.e. the roots of the polynomial $A_c(p)$. As a rule of thumb, the response is acceptable if the poles are located within the shaded area of Figure 3a. The speed of reaction is determined by the distance to the origin, and the slightly oscillatory behavior during settling, by the angle to the negative real axis. The poles closest to the origin will dominate the behavior and are therefore referred to as *dominating poles*.

When discretizing the continuous system,the corresponding discrete-time system $G_d(q)$ will have poles at

$$z_j = e^{s_j T_s}. \tag{8}$$

In addition, the poles will result in a number of zeros, i.e. roots to the numerator polynomial. However, a good approximation for systems of low order is that the characteristics are determined by the poles to $G_d(q)$ and specifications can be stated in the continuous-time domain using (8). Mapping the area in Figure 3a using (8) yields the area in Figure 3b.

Consider the system in Figure 2. The *closed-loop* system is obtained from straightforward computations as (where the argument q has been suppressed)

$$\gamma(t) = \frac{A_g A_r A_F}{A_g A_r A_F + B_g B_r B_F} \left(\frac{B_g B_{ff}}{A_g A_{ff}} \gamma_{tgt}(t) + w(t) + \frac{B_g B_r}{A_g A_r} v(t) \right) \tag{9}$$

From this expression, we see that the design is all about choosing the appropriate polynomials in order to place the poles and zeros at desired locations. Note that the *characteristic polynomial* $P = AA_rA_F + BB_rB_F$ plays an important role. When simplifying expressions like in (9) it is important to remember only to cancel factors that are well-damped, i.e. have roots within the shaded area in Figures 3ab.

A fundamental result from control theory, is that we need an integration in the *open-loop* system in order to drive the error to zero in steady-state. Therefore it may be desirable to introduce integration in the controller, i.e. $(q-1)$ is a factor of A_r.

5 Design

The design will be treated separately for each of the structures in Section 3

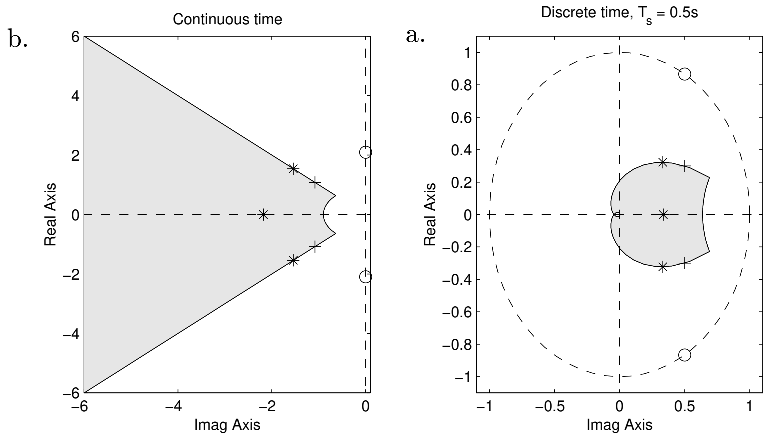

Figure 3: Poles located in the shaded area corresponds to acceptable behavior of the system in continuous time (a) and discrete time (b). The 'o':s correspond to the use of the DPC algorithm, the '+':s to an optimized I-controller, and the '*':s to an optimized PI-controller. In this case no smoothing filter is considered ($\lambda = 0$).

5.1 I-controller

In this case, the characteristic polynomial is given by

$$P(q) = q^2 - (1+\lambda)q + K_i T_s (1-\lambda),$$

which is easy to solve in terms of λ. Assume that we prioritize fast responses, the corresponding dominating continuous-time poles should be as distant as possible from the origin and within the shaded area. The resulting pole locations when $\lambda = 0$ are found in Figure 3. For comparison, the poles when employing the DPC algorithm are plotted.

5.2 PI-controller

Since we now have two degrees of freedom, we expect to be able to do more. The characteristic polynomial is given by

$$\begin{aligned} P(q) = &q^3 - (1+\lambda)q^2 + \left((K_p + K_i T_s)(1-\lambda) - \lambda\right) q - \\ &K_p(1-\lambda). \end{aligned} \tag{10}$$

For fastest possible response, we require to place the corresponding continuous-time poles at

$$s = r_0 \left[e^{i\pi(1-\delta)},\, e^{i\pi},\, e^{i\pi(1+\delta)} \right], \tag{11}$$

where r_0 is the distance to the origin and $0 \le \delta \le 0.25$ is a design parameter describing the angle to the negative real axis. Using Equations (8) and (11), the corresponding desired characteristic polynomial in discrete time is obtained. After some simplifications and a comparison with (10), we see that the optimal distance to the origin is obtained by (numerically) solving the following equation for r_0

$$e^{-r_0 T_s} + e^{-r_0 T_s \cos(\pi\delta)} \cos\left(r_0 T_s \sin(\pi\delta)\right) - 1 - \lambda = 0$$

The controller parameters are then computed as

$$\begin{aligned} K_p = &\frac{1}{1-\lambda} \left(e^{-r_0 T_s (1 + 2\cos(\pi\delta))} \right) \\ K_i = &\frac{1}{T_s(1-\lambda)} \Big[\lambda + e^{-2 r_0 T_s \cos(\pi\delta)} + \\ &e^{r_0 T_s (1+\cos(\pi\delta))} \cos\left(r_0 T_s \sin(\pi\delta)\right) \Big] - \frac{K_p}{T_s} \end{aligned}$$

The resulting pole locations are found in Figure 3 for the case $\lambda = 0$.

5.3 General Linear Controller

The polynomials A_{ff} and B_{ff} will be discussed further in the end and are treated as equal to unity initially. In the general case, we are able to place the close-loop poles at arbitrary locations. However, when the order of the system is higher, the zeros of the system, will affect the behavior. Thus, we have to place the zeros as well.

Consider the characteristic polynomial from (9)

$$\begin{aligned} P(q) =&(1-\lambda)B_r(q) + q(q-\lambda)A_r(q) = \\ &B(q)B_r(q) + A(q)A_r(q). \end{aligned} \tag{12}$$

The design problem breaks down to find A_r and B_r so that the left side of (12) is equal to the specified $P(q)$. This *Diophantine equation* [4] has a solution if

$$\begin{aligned} \deg\{B\} &\leq \deg\{A\} \\ \deg\{A_r\} &\geq \max(\deg\{A\}-1, \deg\{B\}) \\ \deg\{B_r\} &= \deg\{A\}-1 \end{aligned}$$

Note that if A_r^0 and B_r^0 is a solution, so is

$$A_r = A_r^0 + QB,\ B_r = B_r^0 - QA,$$

where $Q(q)$ is a general polynomial. Integration is easily employed by introducing

$$A_r = (q-1)\tilde{A}_r,\ \tilde{A} = (q-1)A,$$

and solve $P(q) = B(q)B_r(q) + \tilde{A}(q)\tilde{A}_r(q)$ instead of (12).

In the design, start by designing A_r and B_r for acceptable noise attenuation of $v(t)$ and the fast variations in $w(t)$ that we cannot track. Then use the polynomials A_{ff} and B_{ff} to shape the transfer function for tracking $\gamma_{tgt}(t)$. Remember to only cancel factors corresponding to *well damped roots* (i.e. in the shaded area in Figure 3.

5.4 Discussion

It may seem appealing to be able to place the poles arbitrarily, but it is important to note that the many degrees of freedom also is a curse in the design procedure, since it also involves placing zeros. When complexity is an issue, the simplicity of the PI-controller is appealing and it is often sufficient. However, when statistics of the disturbances are well known, this can more extensively be used in the design of a controller of higher order.

6 Simulations

The performance of different controller structures are illustrated using simple global network simulations. The scenario reflects the reactions of four mobile stations with settled powers, when a fifth mobile station establishes a call, initially using maximum power. No noise $v(t)$ is applied, the gains g_{ij} are constant, and the mobiles remain fixed throughout the simulations. The I- and

PI-controllers in Figure 3b both have poles in the shaded area, but as indicated by the simulations in Figure 4, the PI-controller corresponds to a more appealing response.

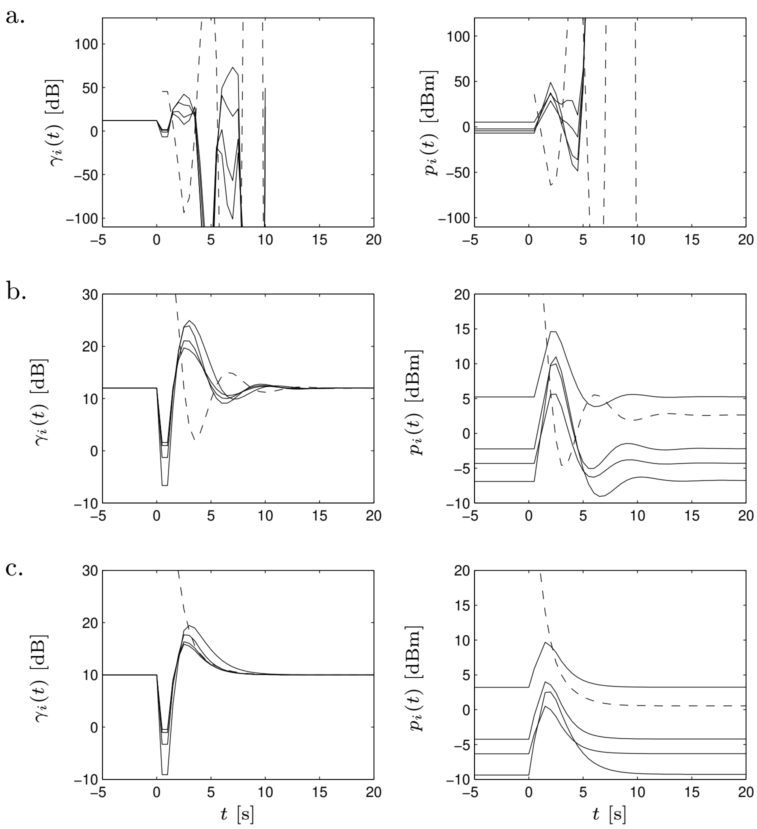

Figure 4: CIR (left) and output powers (right) from simulations. a) DPC algorithm, b) I-controller, c) PI-controller.

Furthermore, the different algorithms were employed in a more comprehensive simulator (see [12] for some and [5] for more details). The signal gains were affected by fading (slow and fast), and the 27 channels were distributed in a 3/9 reuse pattern. Pseudo-random frequency hopping was applied, reducing the effects of fast fading. A sample interval of 0.48s was used; same as the power control interval in GSM. The quality was measured using C/I, and a user is considered satisfied when having a mean C/I above $\gamma^* = 10$ dB. As seen in Figure 5, the PI-controller is again the most appealing choice.

7 Conclusions

Time delays reduces the performance of any controlled system. If neglected in the design phase, the system may even be unstable using the designed controller.

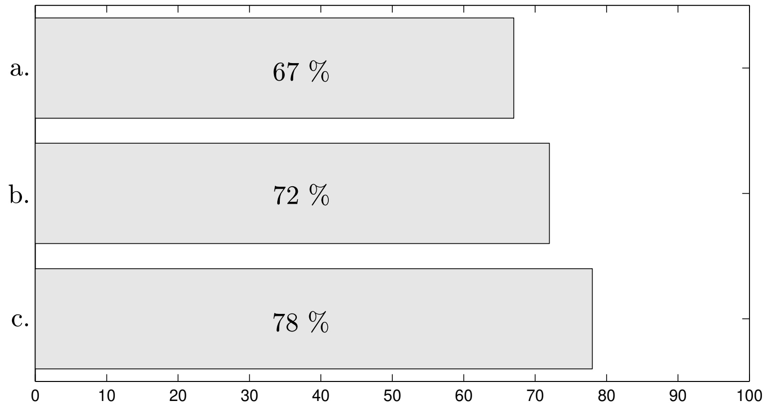

Figure 5: The percentage of satisfied users. a) An I-controller, b) an optimized I-controller, c) an optimized PI-controller.

In this work, techniques from the field of automatic control, such as pole placement, are reviewed. Time delays when identified are naturally considered in the design phase. The pole placement is more intuitive in continuous-time, and using a pole transformation from continuous to discrete-time poles, this intuition is utilized for performance specifications.

Three basic controller structures have been discussed in operation with an optional smoothing filter. Most discussed algorithms can be seen as special cases of an integrating controller and this is therefore a first choice. The natural extension to a PI-controller is discussed, as well as a general linear controller of higher order. In most cases, the PI-controller represents a relevant trade-off between performance and complexity. However, when the statistics of the disturbances is well known, a more general structure is required.

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