F2E5216/TS1002 Adaptive Filtering and Change Detection

Fredrik Gustafsson (LiTH) and Bo Wahlberg (KTH)

Lecture 2

- Basics of theory and examples
- The signal estimation problem
- Applications and simple algorithms

Review Lecture 1

Given: Data $x[n]$ with known distribution under the *null hypothesis* H_0 and the *alternative hypothesis* H_1, respectively.

General test: Form a test statistic $T(x)$.

$$\text{Decide } H_0 \quad \text{if} \quad T(x) < \gamma \text{ (threshold)}$$
$$\text{Decide } H_1 \quad \text{if} \quad T(x) > \gamma$$

- *Probability of false alarm* $P_{FA} = P(H_1|H_0) = \int_{x:T(x)>\gamma} p(x|H_0)dx$
- *Probability of detection* $P_D = P(H_1|H_1) = \int_{x:T(x)>\gamma} p(x|H_1)dx$

Likelihood Ratio Tests

$$\textbf{LRT}: \; T(x) = \frac{p(x|H_1)}{p(x|H_0)}, \quad \text{PDF: } g(t|H_i)$$

$$\textbf{GLRT}: \; L(x) = \frac{p(x|H_1, \hat{\theta}_1^{ML})}{p(x|H_0, \hat{\theta}_0^{ML})}, \quad \text{(asymptotic) PDF: } g(l|H_i)$$

Threshold:

$$P_{FA} = \int_\gamma^\infty g(s|H_0)ds = \alpha \;\Rightarrow$$
$$P_D = \int_\gamma^\infty g(s|H_1)ds$$

Basic Theory

A 30 minutes primer on:

- Signal estimation
- Adaptive filtering
- Kalman filtering
- Change detection
- Evaluation

Accompanying texts: Gustafsson, Chapter 1-2.

Signal estimation

Signal model

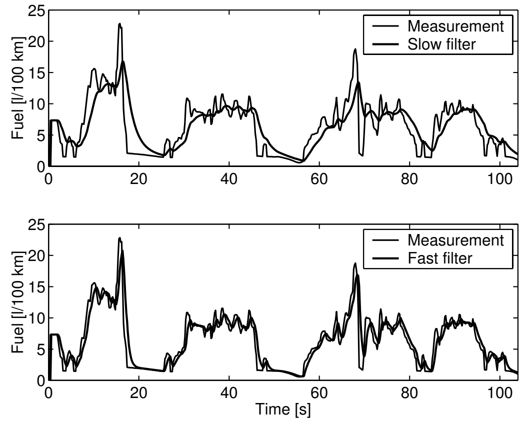
$$y(t) = \theta(t) + e(t)$$

Common algorithms:

$$\begin{aligned}\hat{\theta}(t+1) &= \hat{\theta}(t) + \gamma\varepsilon(t)\\ \varepsilon(t) &= y(t) - \hat{y}(t),\end{aligned}$$

$$\hat{\theta}(t+1) = (1-\lambda_t)\hat{\theta}(t) + \lambda_t\varepsilon(t) \quad \text{(forgetting factor)}$$

Example: fuel consumption estimation.



Slow filter $\Rightarrow$ good noise attenuation. Fast filter $\Rightarrow$ good tracking.

Parameter estimation using adaptive filtering

Parametric model of linear system

$$y(t) = G(q;\theta)u(t) + H(q;\theta)e(t)$$

Special consideration to models linear in parameters (ARX etc):

$$y(t) = \varphi^T(t)\theta + e(t)$$

Generic adaptive filter:

$$\begin{aligned}\hat{\theta}(t+1) &= \hat{\theta}(t) + \gamma K(t)\varepsilon(t)\\ \varepsilon(t) &= y(t) - \hat{y}(t),\end{aligned}$$

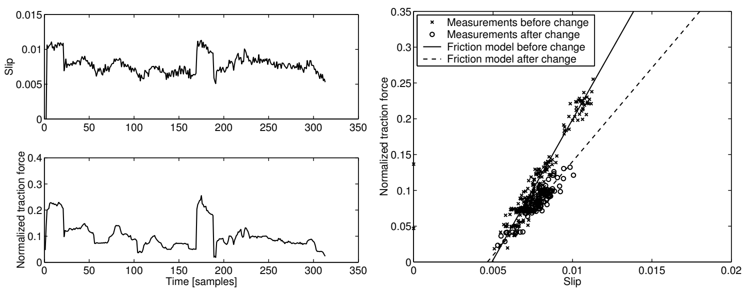
RLS, WLS, LMS and KF correspond to different $K(t)$.

Example: friction estimation

$$y_t = \theta_t^{(1)}u_t + \theta_t^{(2)} + e_t,$$

θ_t contains a slope and an offset,
u_t is the input to the model (the engine torque)
y_t is the measured output (the so called wheel slip).
For friction monitoring, the slope $\theta_t^{(1)}$ is the relevant parameter

Change detection = modeling and estimation in order to map the problem on a standard detection test



State estimation using Kalman filtering

State space model:

$$\begin{aligned} x(t+1) &= Ax(t) + B_u u(t) + B_v v(t) + \delta(t-k) B_f f(t) \\ y(t) &= Cx(t) + D_u u(t) + D_e e(t) + \delta(t-k) D_f f(t) \end{aligned}$$

Controlled and measured inputs: $u(t)$

Process noise: $v(t)$

Measurement noise: $e(t)$

Faults in actuator and sensor, or a state disturbance: $f(t)$

The Kalman filter

$$\begin{aligned} \hat{x}(t+1) &= A\hat{x}(t) + K(t)\varepsilon(t) \\ \varepsilon(t) &= y(t) - \hat{y}(t) = y(t) - C\hat{x}(t), \end{aligned}$$

The residual $\varepsilon(t)$ can be used for change detection.

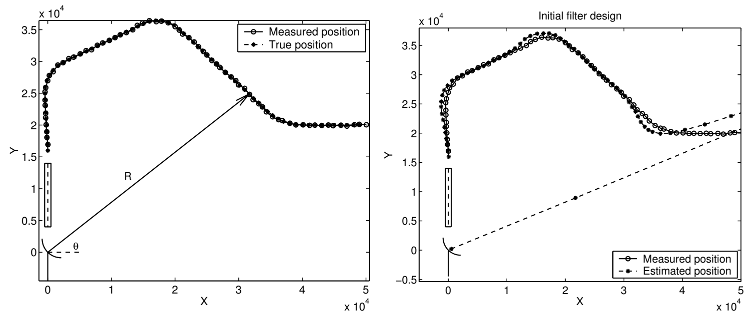
The Kalman filter is optimal for *Gaussian noise* in the senses of:

Minimum variance estimator: There is no other estimator that gives smaller variance error Var$(\hat{x}(t) - x(t))$

Conditional expectation of $x(t)$, given the observed values of $y(t)$.

Tuning the Kalman filter

Compromise between tracking ability and noise attenuation is controlled by the signal to noise ratio (SNR) Var $(v(t))/$ Var $(e(t))$, which is tuned by the scalar γ.



Increase the tracking ability by increasing the SNR.
Rather poor result, and in particular in the transient!

An algebraic alternative

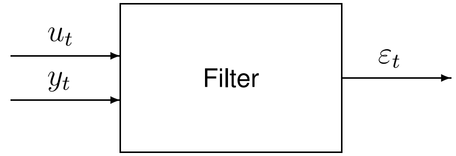
Parity space: Project vectors of stacked measurements onto a subspace, which is defined as the residual.

$$\varepsilon(t) = W^T(Y - H_u U)$$

With proper design of W, the residual will react to certain faults in specific patters, making fault isolation possible.

Summary of filtering

For change detection, the filter can be seen as a residual generator:



Change detection approaches:

Whiteness test

Under H_0, ε_t is i.i.d. and $N(0, 1)$.

Define a distance measure s_t from i.i.d., for example $s_t = \varepsilon_t$, $s_t = \varepsilon_t^2 - \lambda$ and $s_t = K_t \varepsilon_t$ (vector with correlations).

Then

$$H_0 : s_t = w_t$$
$$H_1 : s_t = A + w_t$$

and we are back to detection theory.

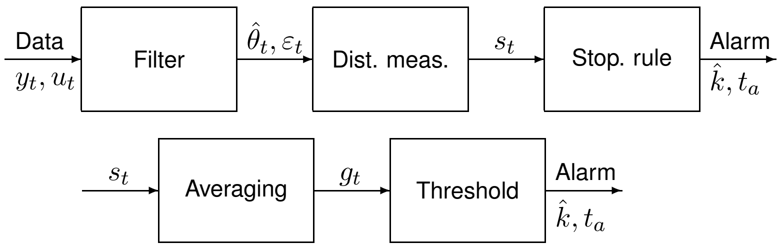
Change detection approaches, cont.

Parallel filter approach where one slow (global) and one fast filter are compared.

Filter bank based on a multiple hypothesis test.

Stopping rules

Change detection based on a hypothesis test for no change and change respectively, needs a stopping rule. This includes both the whiteness test and parallel filter approaches.



Example: GLRT $g_t = \frac{1}{N}\sum_{t=1}^{N} s_t \leftrightarrow T(x)$ for batchwise processing

Examples of on-line stopping rules

CUSUM : $g_t = \max(g_{t-1} + s_t - \nu, 0)$

WMA: : $g_t = \sum_{k=t-L+1}^{t} s_k$

GMA : $g_t = \lambda g_{t-1} + (1-\lambda) s_t$

Alarm if $g_t > h$

More or less low-pass filtering and thresholding.

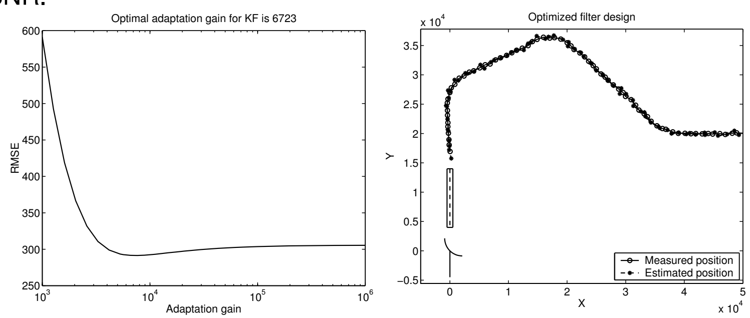
Evaluation and Design

- **For surveillance**, tracking ability and variance error in the estimates are the main performance measures.

- **For fault detection,** it is of importance to get the alarms as soon as possible – the delay for detection – while the number of false alarms should be small.

Example on tracking design

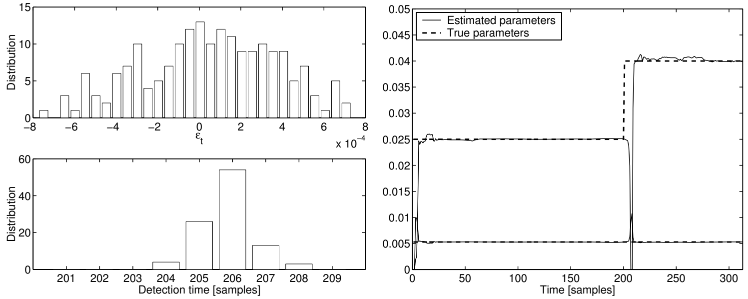
The Kalman filter for target tracking has a unique minimum for its SNR.



Better transient due to large initial error covariance matrix

Example on tracking and change detection design

Evaluation of a particular adaptive filter for friction estimation:



Signal Estimation and Surveillance

Today:

- The change in the mean problem (signal estimation)
- Some applications
- Averaging, filtering and estimation approaches
- Stopping rules

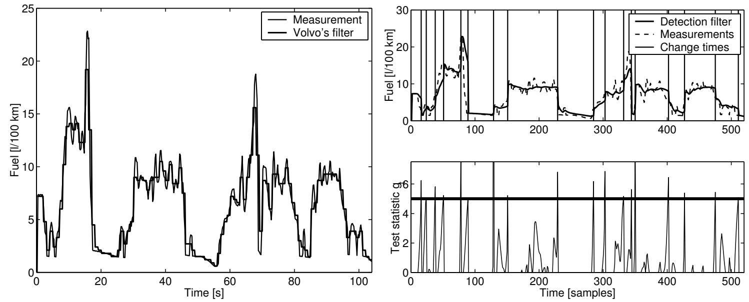
Change in the mean model

$$y_t = \theta_t + e_t, \ \ \mathrm{E}(e_t^2) = \lambda$$

Problems:

- Monitoring of θ
- Limit checking of θ
- Detection of abrupt changes in θ
- Detection of abrupt changes in λ

Application: Fuel Monitoring

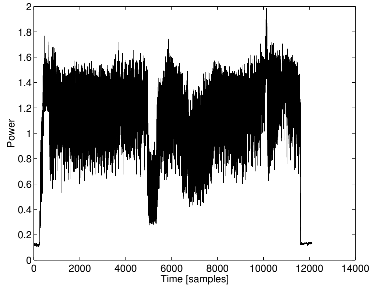


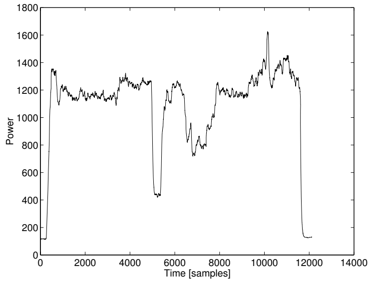
Improve Volvo's filter (solid line to the left) with respect to:

- *Noise attenuation*
- *Tracking speed at abrupt accelerator changes*

Surveillance of θ should be approached with change detection ideas!

Application: Paper refinery

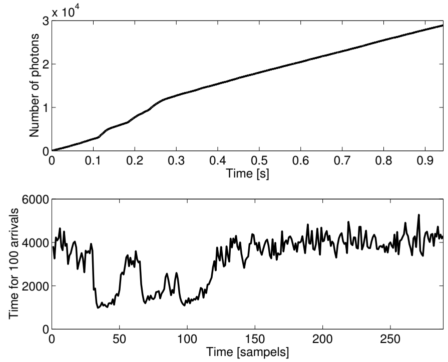




Power signal from grinding engine:

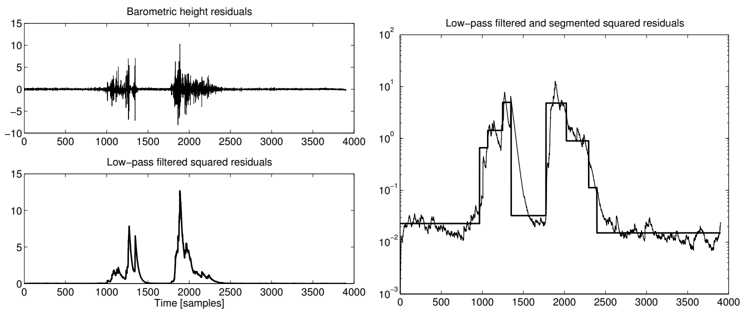
- The noise must be considerably attenuated to be useful in the feedback loop.
- It is very important to quickly detect abrupt power decreases to be able to remove the grinding discs quickly and avoid an expensive disc crash.

Application: photon arrival



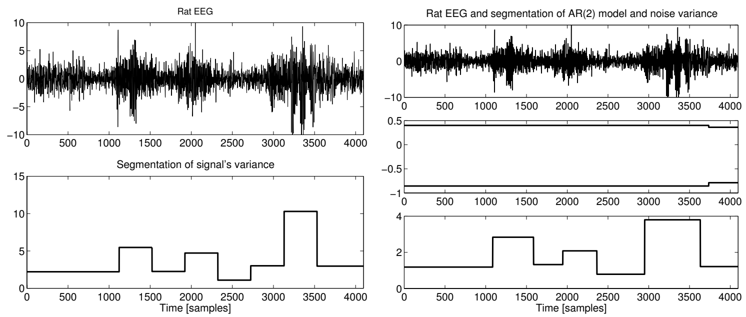
Poisson process with piecewise constant arrival intensity.
Problem: Tracking the brightness changes of galactical and extragalactical objects

Application: altitude sensor



Sensor variance changes during transonics.
Problem: Change detection and modeling of noise variance.

Application: Rat EEG



Activity or background noise can be classified using variance change detection.

Exercises for Lectures 2 and 3

Link on homepage

http://www.control.isy.liu.se/~fredrik/detect/exercises.pdf

Exercise: 4, 5, 6, 8, 9, 10, 13

Next Time

Change detection methods for change in the mean:

- The CUSUM test
- Filter and detector evaluation
- The likelihood concept
- Maximum likelihood and likelihood ratio based CD
- Information based CD